**14th International Conference on Multiphase Flows in Industrial Plant , MFIP17**

# New insights into the modeling of stratified flow characteristics and stability boundaries

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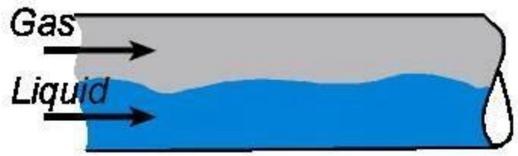
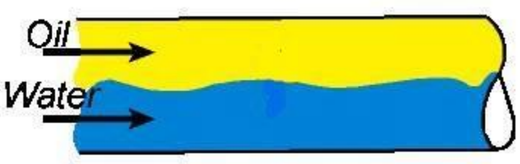
# Motivation

**Stratified flow** is a basic flow pattern in gas-liquid and liquid-liquid systems; It is frequently encountered in various important industrial processes (e.g., gas-condensate pipelines operate primarily in the stratified flow regime).




## Basic information for engineering applications:

Pressure gradient, holdup (in horizontal and inclined channels and pipes).

Region of existence (i.e., stability)

**Modeling Complexity-** interaction between the phases, gravity effects, surface tension effects, waves, turbulence, non-unique solution for specified operational conditions….

# Modeling Approach

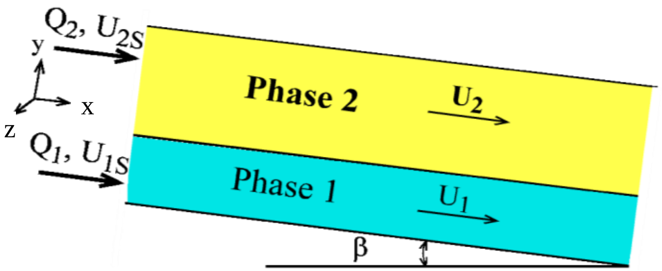
- Exact solutions - only for laminar flows.
- Numerical solutions (CFD)
- Mechanistic models: 1-D Two-Fluid Models.

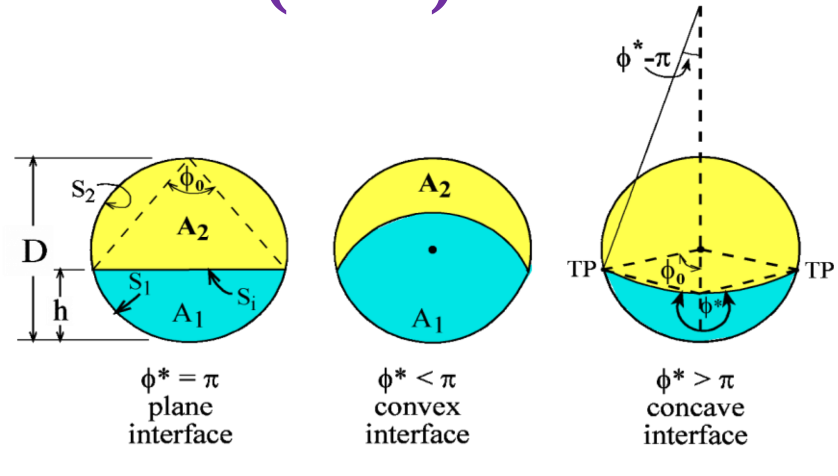
Exact solution for laminar flows are applicable for liquid-liquid systems and small diameter pipes.

Exact solutions: useful as benchmark for numerical codes, for testing closure relations for the Two-Fluid models, starting point for rigorous stability analysis.

What insight can be gained from the exact solutions that may improve the modeling of the stratified flow characteristics and its stability boundaries??

# Fully Developed Laminar Pipe Flow (LPF)- Exact solution



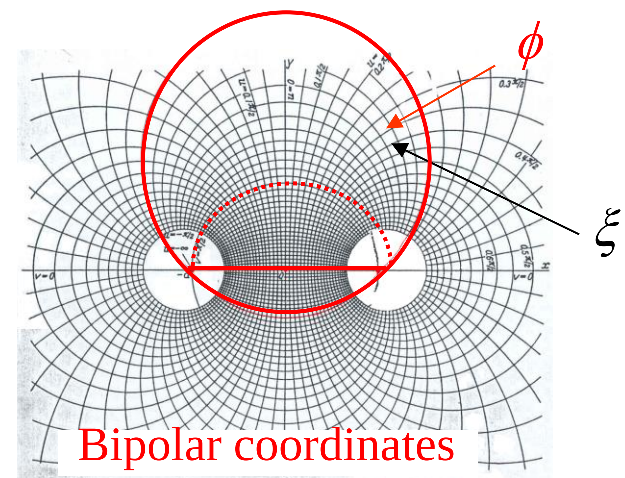




2-D velocity profiles: $u_1(y,z)$, $u_2(y,z)$ *in the form of Fourier Integrals:*

*1≡Heavy phase, 2 ≡Light phase*

$$\mu_1\left(\frac{\partial^2 u_1}{\partial y^2}+\frac{\partial^2 u_1}{\partial z^2}\right)=\frac{dp}{dx}-\rho_1 g\sin\beta$$

$$\mu_2\left(\frac{\partial^2 u_2}{\partial y^2}+\frac{\partial^2 u_2}{\partial z^2}\right)=\frac{dp}{dx}-\rho_2 g\sin\beta$$




Given the interface location
(i.e. holdup and curvature - $\phi^*$)

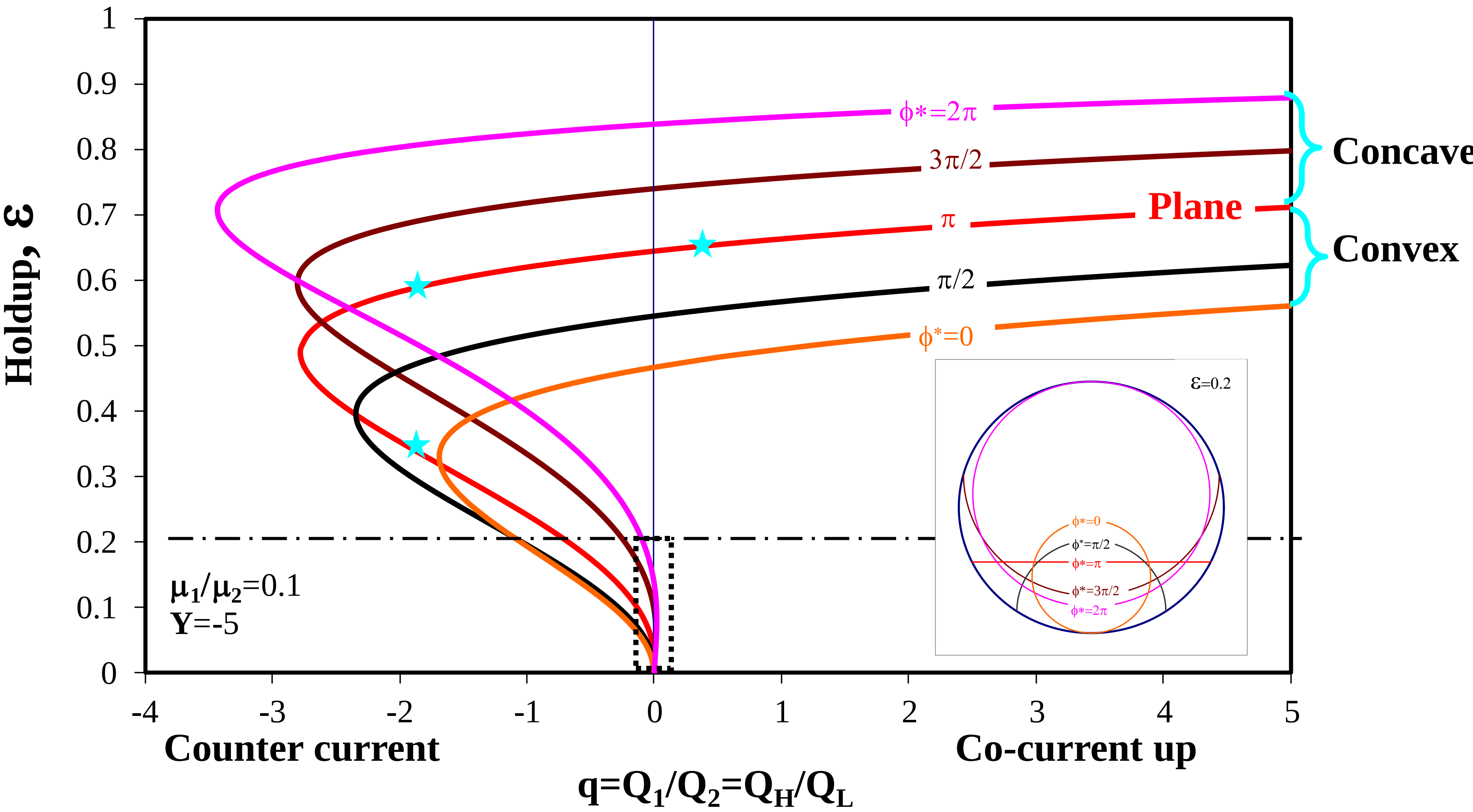
**Boundary Conditions:**

- No-Slip condition at the pipe wall
- Continuity of the velocities and tangential shear stress across the phases interface.

# Stratified LPF- Upward Inclined Pipes- Curved Interface

*Goldstien, Ullmann & Brauner, IJMF (2015)*

Holdup=$\varepsilon(q, m, Y, \phi^*)$ $\quad m=\mu_1/\mu_2$; $\quad Y=\dfrac{((\rho_1-\rho_2)g\sin\beta)}{(-dp_f/dz)_{2S}}$; $\quad (-dp_f/dz)_{2S}=\dfrac{8Q_2\ \mu_2}{\pi R^4}$



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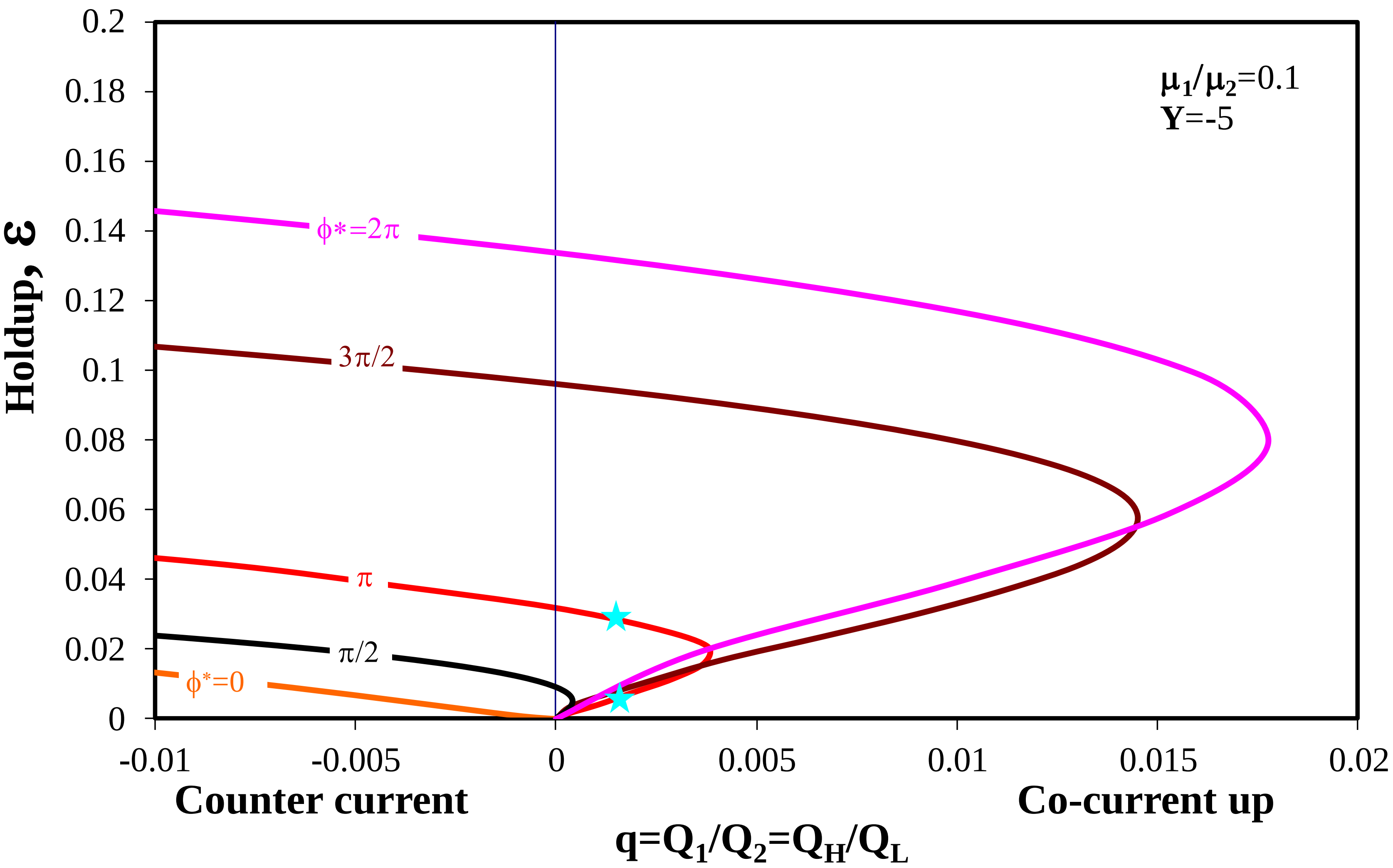
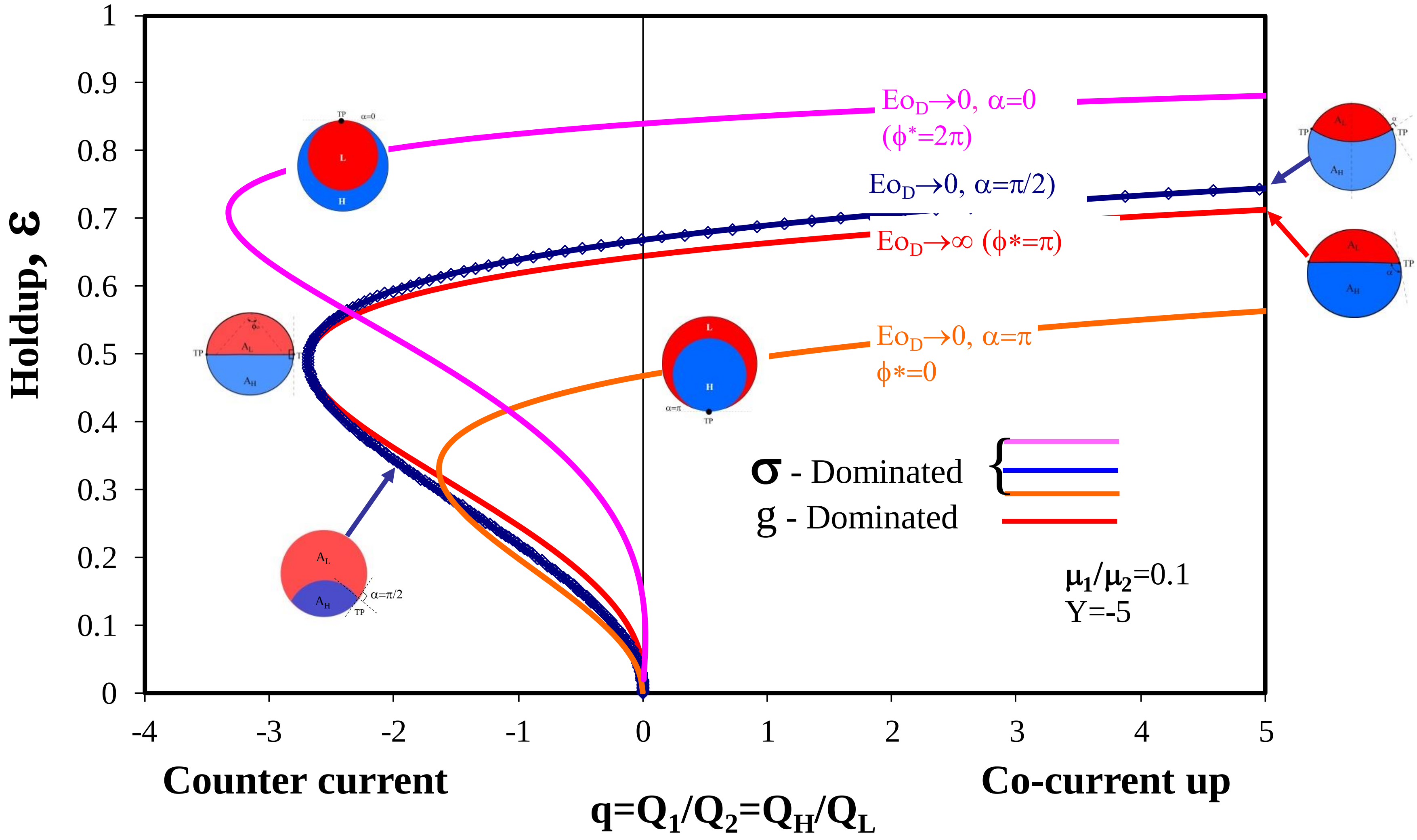


# The effect of contact angle, α and $Eo_D = D^2 \Delta\rho g \cos\beta / \sigma$

$$\phi^* = f(\varepsilon, \alpha, Eo_D)$$

*(Gorelik and Brauner, 1999)*

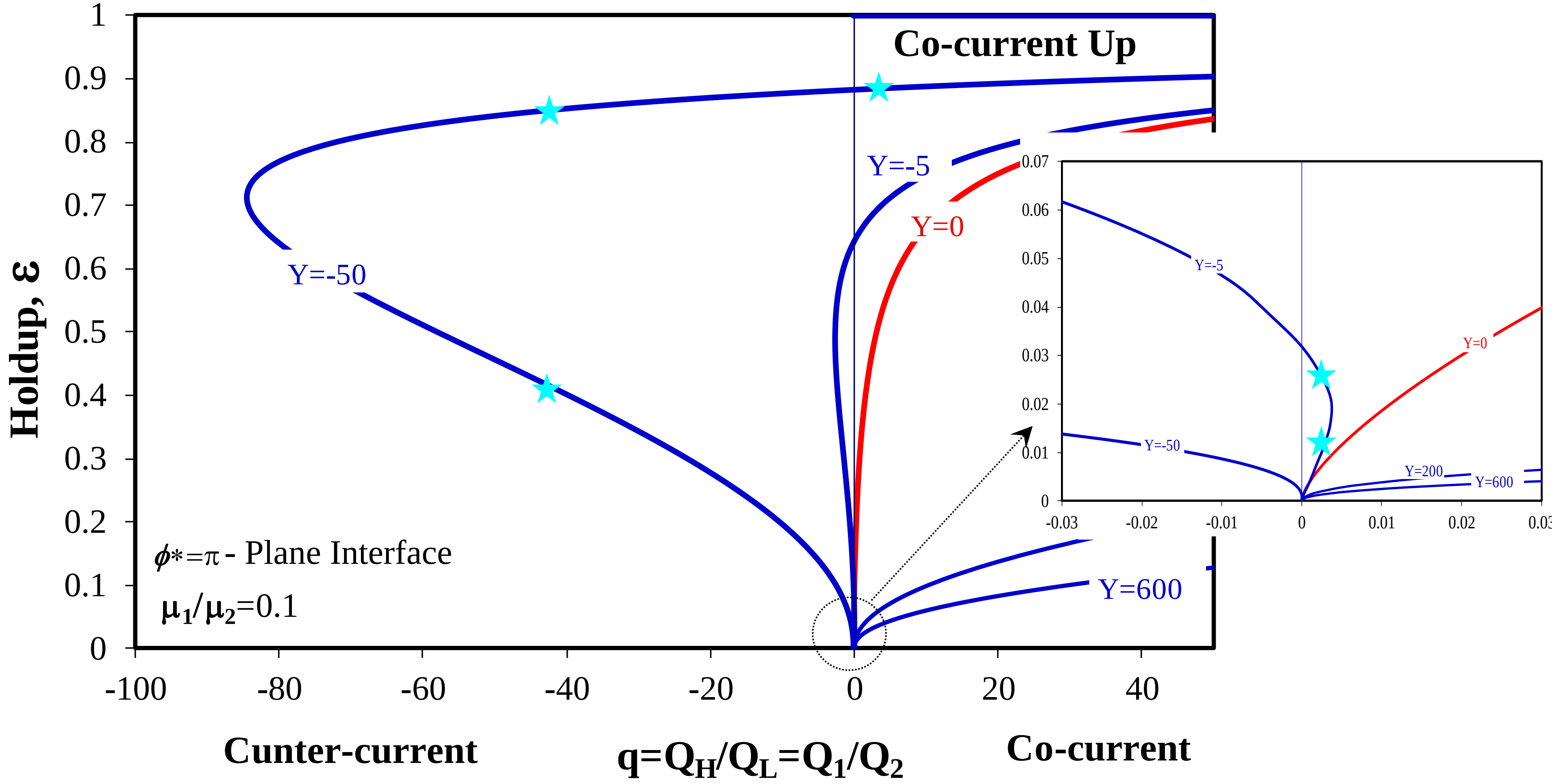
$$\varepsilon = f(\tilde{\mu}, q, Y, Eo_D, \alpha)$$



# Stratified LPF - Effect of Inclination- Plane interface

$$\varepsilon(q, m, Y, \phi^* = \pi) = 0$$

$$q = \frac{Q_H}{Q_L} = \frac{Q_1}{Q_2} \qquad m = \frac{\mu_H}{\mu_L} = \frac{\mu_1}{\mu_2} \qquad Y = \frac{((\rho_1 - \rho_2) g \sin\beta)}{(-dp_f/dz)_{2S}} \qquad (-dp_f/dz)_{2S} = \frac{8Q_2\,\mu_2}{\pi R^4}$$



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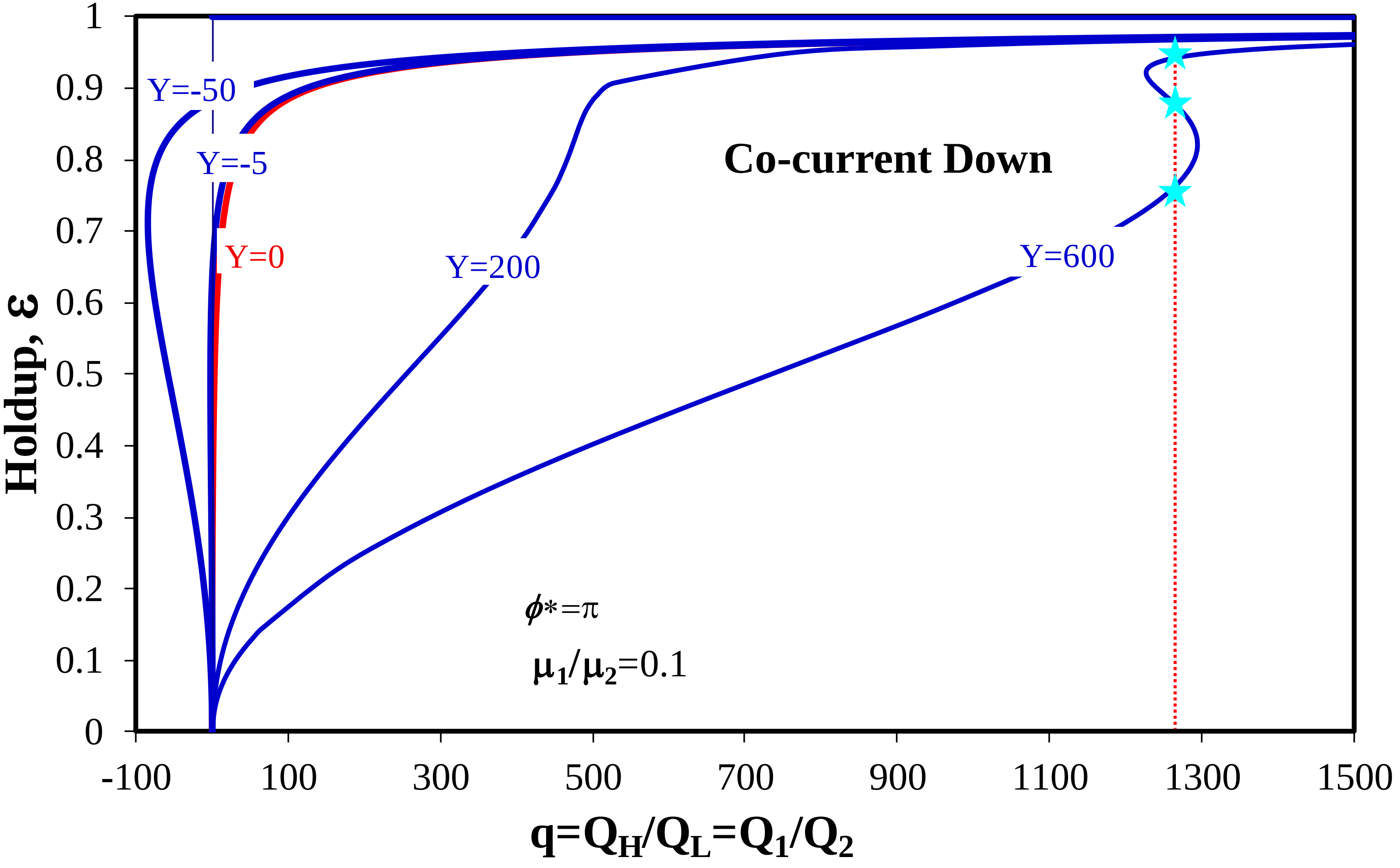
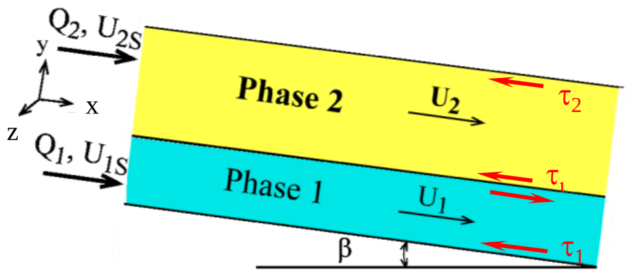


# Mechanistic Models – Two-Fluid model (TF)

$$U_{1s,2s} = \frac{Q_{1,2}}{A};$$




Mass Conservation $\bar{U}_1 = \frac{U_{1s}}{\varepsilon}; \qquad \bar{U}_2 = \frac{U_{2s}}{1-\varepsilon}$

Momentum Equations

$$-A_1 \frac{dp}{dx} + \tau_1 S_1 - \tau_i S_i + \rho_1 A_1 g \sin\beta = 0$$

$$-A_2 \frac{dp}{dx} + \tau_2 S_2 + \tau_i S_i + \rho_2 A_2 g \sin\beta = 0$$

Required Closures:

Interfacial curvature: $\phi^*$ (e.g., plane interface)

Wall Shear Stresses: $\tau_1; \quad \tau_2$

Interfacial Shear: $\tau_i$

# Theory based closures for the shear stresses

**M**odified **T**wo-**F**luid (**MTF** model, *Ullmann & Brauner, IJMF, 2004, 2006*)

## Wall Shear

$$\tau_{1,2} = -\frac{1}{2} f_{1,2}\rho_{1,2} \left|\bar{U}_{1,2}\right| \bar{U}_{1,2} \cdot F_{1,2}$$

e.g., laminar

$$f_{1,2} = \frac{16}{\mathrm{Re}_{1,2}}$$

Corrections for the Two-phase interactions $F=F(\varepsilon, X^2, U_{1,2}, S_{1,2,i})$

$$F_1 = \frac{1+\frac{\bar{U}_2}{\bar{U}_1}\left[\frac{\tilde{S}_1}{\tilde{S}_1+\tilde{S}_i} X^2\left(\frac{1-\varepsilon}{\varepsilon}\right)^2 - \frac{4\tilde{S}_2}{\pi+2}\right]}{1+\frac{\bar{U}_2}{\bar{U}_1} X^2\left(\frac{1-\varepsilon}{\varepsilon}\right)^2}$$

$$F_2 = \frac{1+\frac{\bar{U}_1}{\bar{U}_2}\left[\frac{\tilde{S}_2}{\tilde{S}_2+\tilde{S}_i} \frac{1}{X^2}\left(\frac{\varepsilon}{1-\varepsilon}\right)^2 - \frac{4\tilde{S}_1}{\pi+2}\right]}{1+\frac{\bar{U}_1}{\bar{U}_2} \frac{1}{X^2}\left(\frac{\varepsilon}{1-\varepsilon}\right)^2}$$

$X^2$- Martinelli parameter

(=$m\cdot$q in laminar flows)

## Interfacial Shear

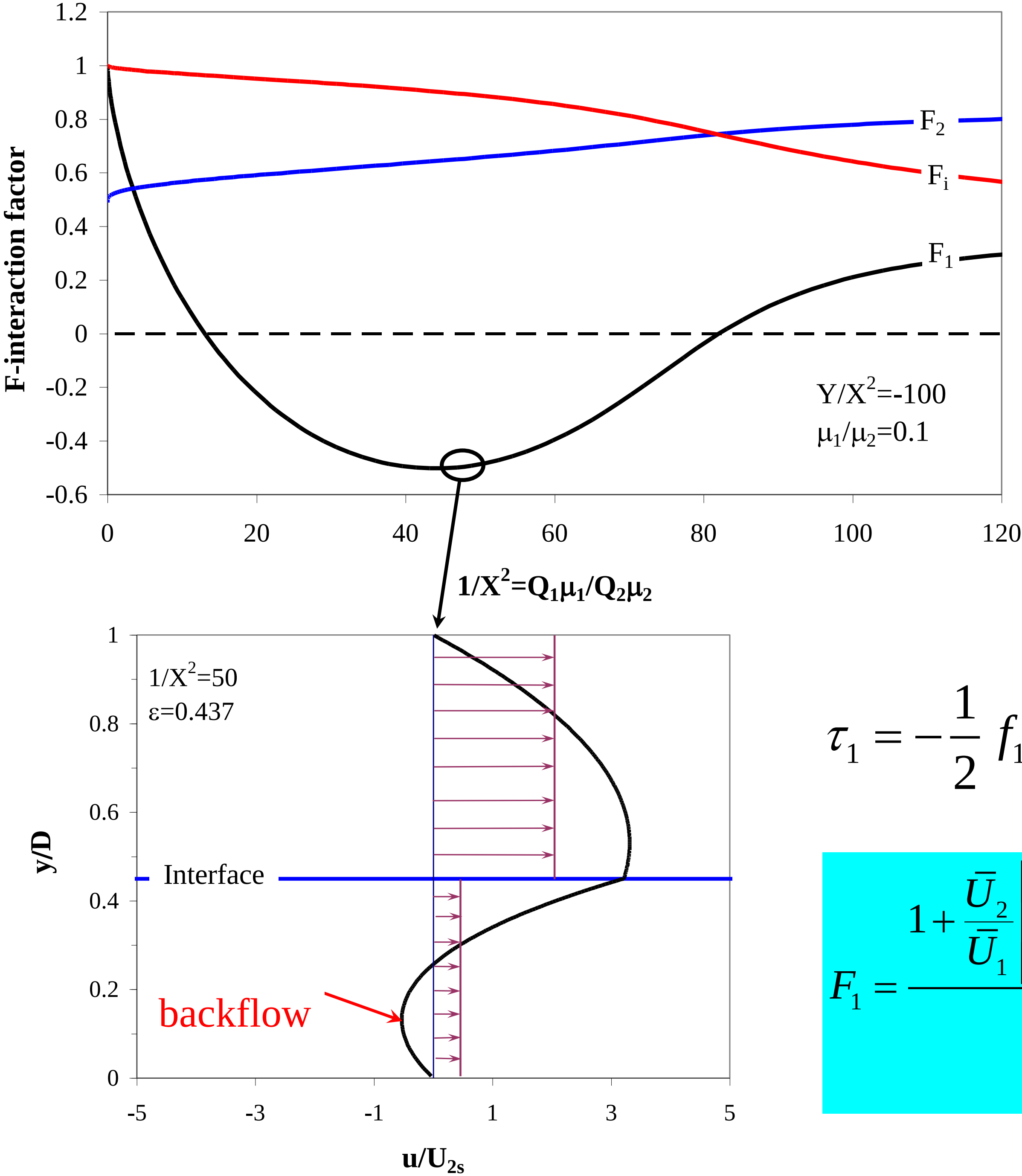
$$\tau_i = \begin{cases} -\frac{1}{2}\rho_1 f_1 |\bar{U}_1|(\bar{U}_2 - \bar{U}_1)\cdot F_{i1} & ;\ \bar{U}_1 > \bar{U}_2 \\ -\frac{1}{2}\rho_2 f_2 |\bar{U}_2|(\bar{U}_2 - \bar{U}_1)\cdot F_{i2} & ;\ \bar{U}_1 < \bar{U}_2 \end{cases}$$

$$F_{i1} = \frac{1}{1+\frac{\bar{U}_2}{\bar{U}_1} X^2\left(\frac{1-\varepsilon}{\varepsilon}\right)^2}$$

$$F_{i2} = \frac{1}{1+\frac{\bar{U}_1}{\bar{U}_2}\frac{1}{X^2}\left(\frac{\varepsilon}{1-\varepsilon}\right)^2}$$

# Co-current up flow:

# Significance of the F-interaction factors in case of backflow

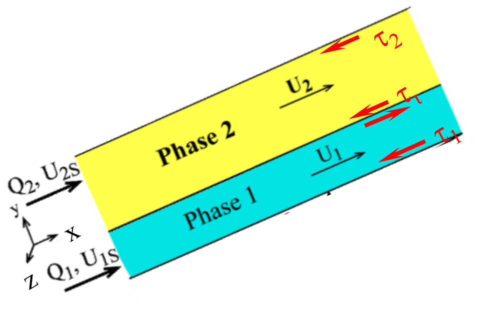


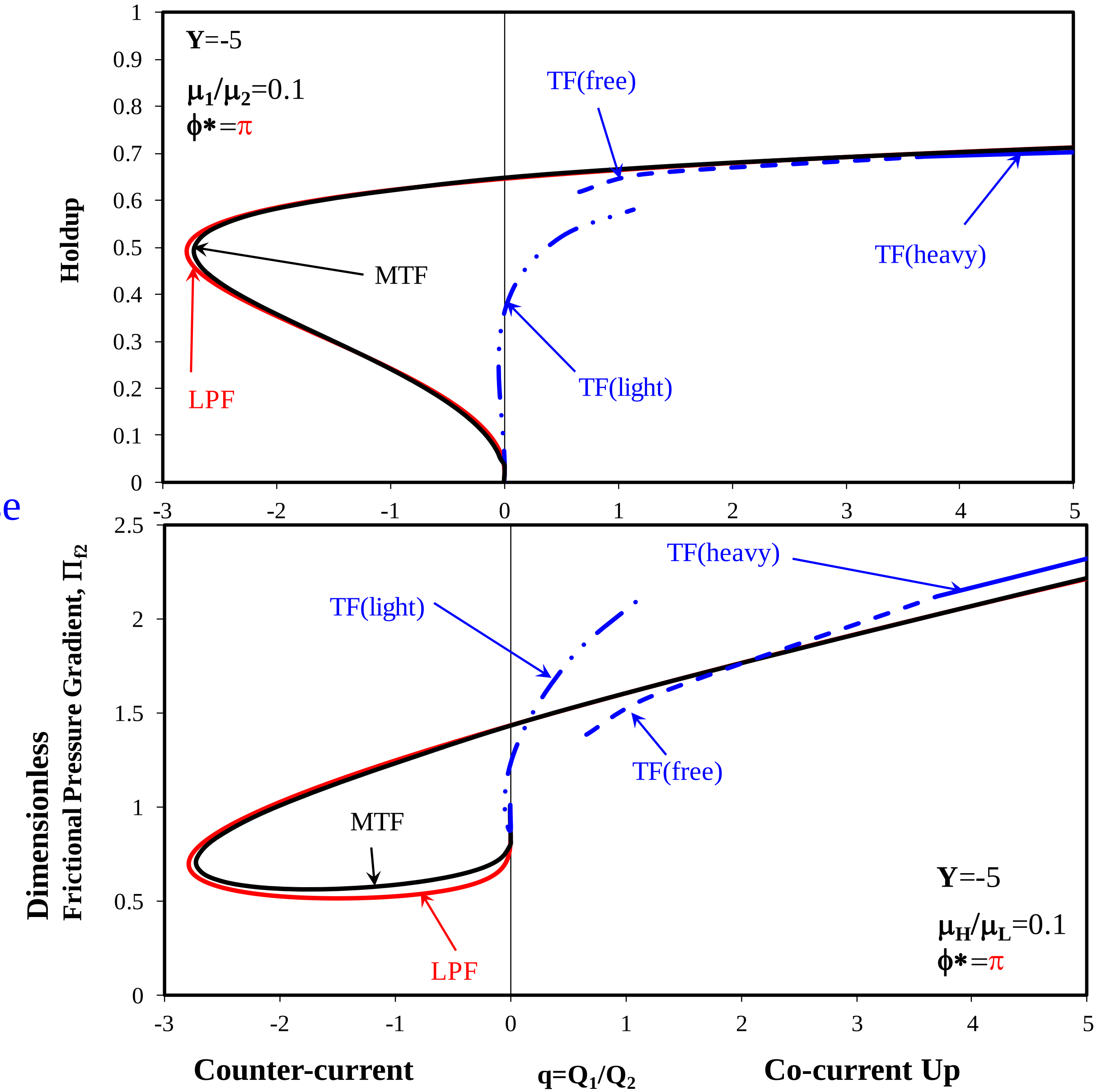

$$\tau_1 = -\frac{1}{2} f_1 \rho_1 \left|\bar{U}_1\right| \bar{U}_1 \cdot F_1$$

$$F_1 = \frac{1 + \dfrac{\bar{U}_2}{\bar{U}_1}\left[\dfrac{\tilde{S}_1}{\tilde{S}_1 + \tilde{S}_i} X^2 \left(\dfrac{1-\varepsilon}{\varepsilon}\right)^2 - \dfrac{4}{\pi + 2}\tilde{S}_2\right]}{\dfrac{\bar{U}_2}{\bar{U}_1} X^2 \left(\dfrac{1-\varepsilon}{\varepsilon}\right)^2}$$

# Stratified Flow- Upward Inclined Pipes

Comparison of the Two-Fluid (TF, MTF) prediction with the exact (LPF) solution




TF (light/heavy)-

$\tau_i$ based on the light/heavy phase

TF (free)-

$\tau_i$ =0 for $\bar{U}_1 \approx \bar{U}_2$

# Some conclusions..

- The Modified TF (MTF) closures- exactly reproduce the solution in the Two-Plate (TP) geometry, and in a good agreement with the exact solution in the pipe geometry for horizontal and inclined laminar stratified flows.

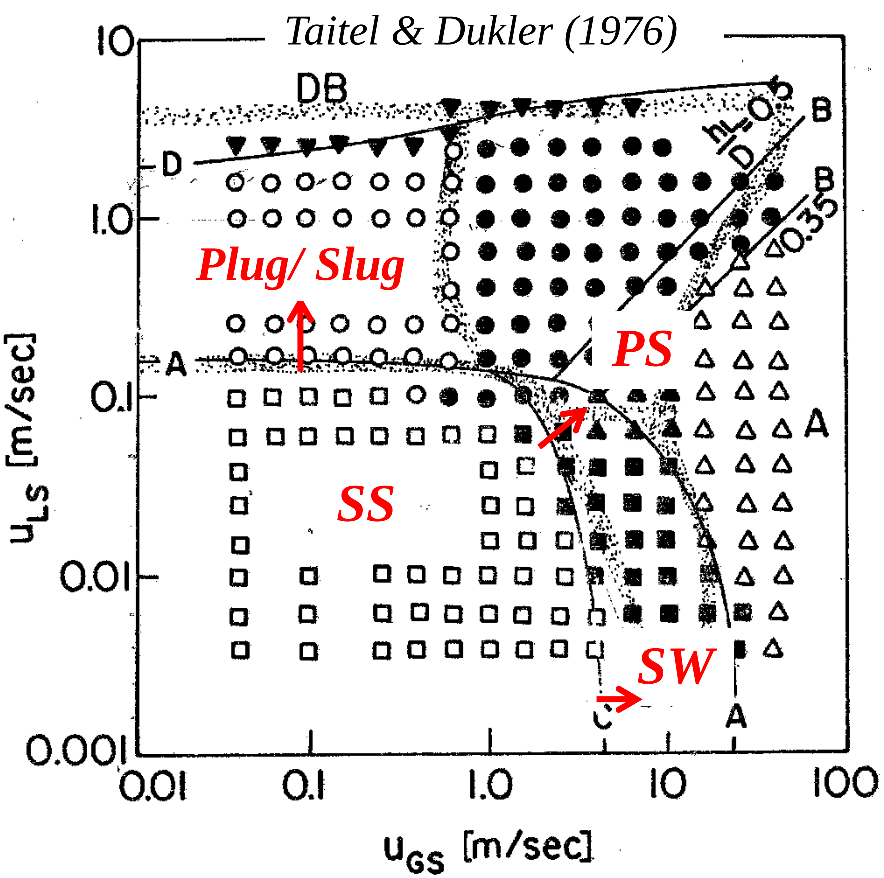
- The MTF closures have been extended for turbulent flows, corrections for the effect of the waves on the shear stresses and interfacial curvature were introduced (*Ullmann & Brauner, IJMF, 2006*).

- Treating the complexity of the problem incrementally enables isolation of the effects of each of the factors and renders more robust closures.

When those models for stratified flow are relevant??

# Stratified Flow Existence Boundaries

e.g., Flow Pattern Map for Air-Water Horizontal System




**Stability analysis** is an essential tool for the prediction of systems parameters and conditions for which **stratified flow is a stable flow pattern**.

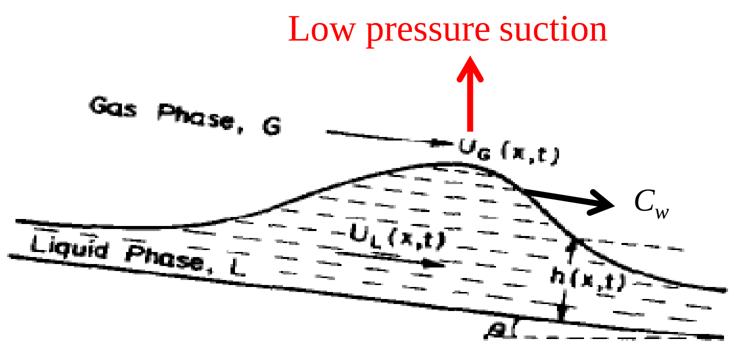
**Exact stability analysis** for two-phase pipe flow is **too complicated.**

# Mechanisms of Wind Generated Waves

A major part of the two-phase flow literature dealt with gas-liquid systems,
The destabilizing mechanisms were related to those responsible for wind-generated waves.

- **Kelvin-Helmholtz (KH) Instability** (1871), **inviscid fluids-** waves grow when suction due to reduced pressure at the wave crest overcome the wave weight.

  The destabilizing force is in phase with the wave height.




- **Jeffreys' 'Sheltering'** (1925), **Miles-Phillips** (1962) theories -

waves grow when the energy input by the wind is larger than the viscous dissipation in the waves.

Wind-Wave interactions result in a destabilizing force that is in phase with the wave slope, which is essential for the *energy transfer.*

$$\tau_G = s\rho_G (U_G - C_w)^2 \partial h / \partial x$$

$$(U_G - C_w)^2 C_w > \frac{4\mu_L g(1-\rho_G/\rho_L)}{s\rho_G}$$

***s*** -"Sheltering coefficient" (s=0.27, tunable)

# Application to Stratified Gas-Liquid Flow in Channels

The classical Kelvin-Helmholtz (KH) instability criterion for inviscid flow (Lamb, 1945) :

$$(U_G - U_L)^2 \geq \frac{1}{k}\left[\tanh(kh_G) + \frac{\rho_G}{\rho_L}\tanh(kh_L)\right]\left[\frac{g(\rho_L - \rho_G)}{\rho_G} + \frac{\sigma}{\rho_G}k^2\right] \qquad k=2\pi/L_w$$

Assuming long waves, k~0 ($L_w >> h_G, h_L$ , $\tanh(kh_G) \simeq kh_G$, , $\tanh(kh_L) \simeq kh_L$) , and $\rho_G / \rho_L << 1$, $U_G >> U_L$:

$$Fr_G = \left[\frac{\rho_G}{(\rho_L - \rho_G)}\right]^{0.5} \frac{\bar{U}_G}{\sqrt{gH}} \geq C\left(1 - \tilde{h}\right)^{0.5}; \quad \tilde{h} = h_L / \mathrm{H}$$

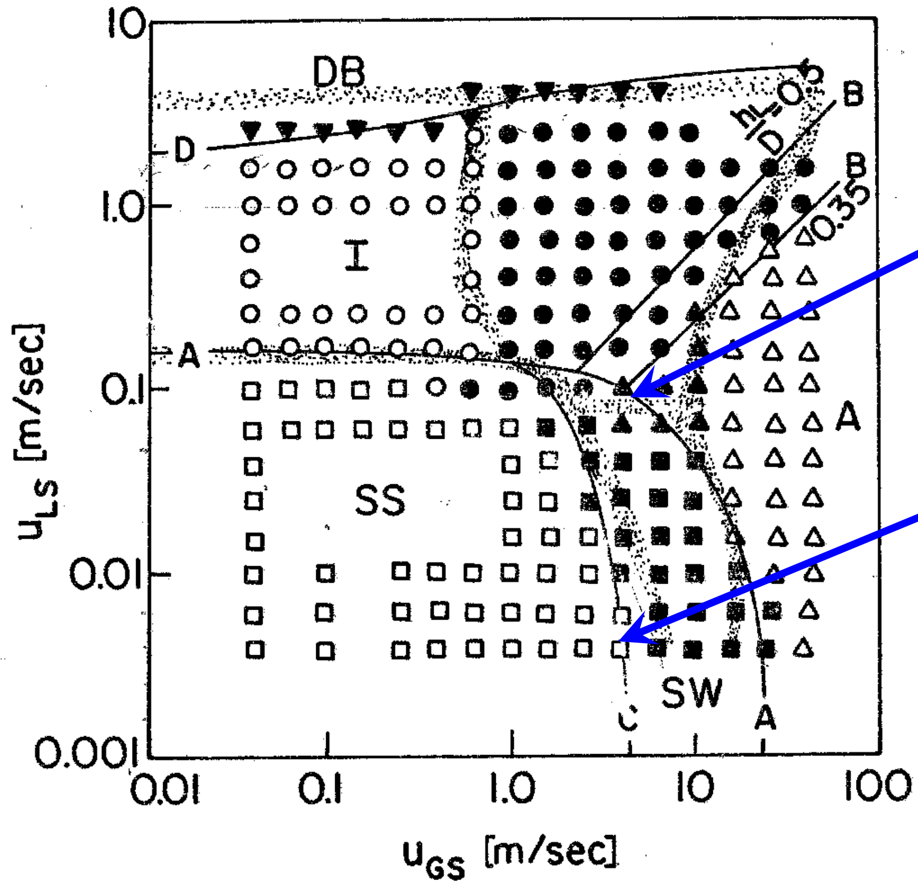
$C=1$ (Kordyban & Ranov, 1970), $C=0.5$ (Walis & Dobson, 1973) - Stratified/ Slug boundary

$C = (1 - \tilde{h})$ (Taitel &Dukler,1976) for Stratified/ non-stratified boundary

- Bernoulli type criterion- the gas suction acts on a "stationary" interfacial disturbance.
- Liquid inertia (heavy phase) is ignored.
- Weak dependence on viscosity (via $\tilde{h}$ calculated by the Two-Fluid SS model).

# Flow Pattern Map: Air-Water Horizontal System

Taitel & Dukler (1976) map , Experimental- Barnea et al. (1980), D=0.025 m




KH mechanism, $C = (1 - \tilde{h})$
Stratified/non-Stratified

Jefreys' 'Sheltering' model,
$s$=0.01 ($s$=0.27)
Stratified Smooth/Stratified-Wavy

**Prediction the of the stratified flow boundary for a general two-phase system requires consideration of the inertia and viscous effects of both phases !**

# Viscid K-H Stability Analysis

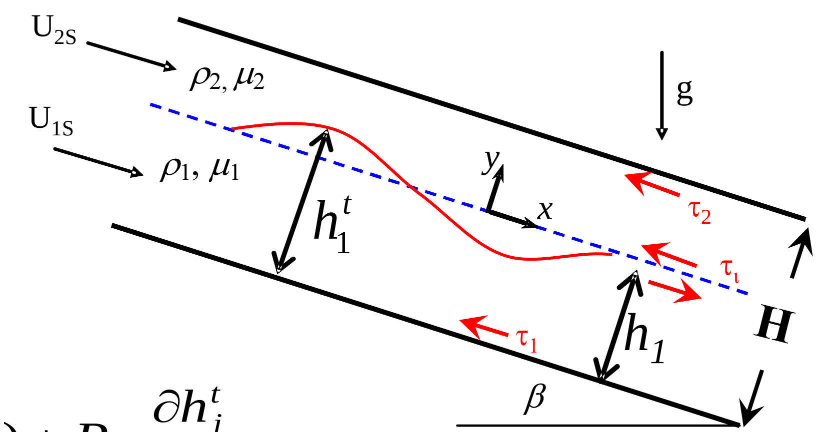
## Transient Two Fluid Model for Stratified Flow

e.g., Lin & Hanratty (1986), Andritsos et al., (1989), Barnea, (1991), Brauner & Moalem, (1991,1992), Barnea & Taitel, (1993)…..

### Continuity Equations

$$\frac{\partial}{\partial t}(\rho_j h_j^t) + \frac{\partial}{\partial x}(\rho_j h_j^t \bar{u}_j) = 0 \ ;$$

**$j$=1, heavy**
**$j$=2, light**




### Momentum Equations

$$\frac{\partial}{\partial t}(\rho_j h_j^t \bar{u}_j) + \frac{\partial}{\partial x}(\rho_j h_j^t \gamma_j \bar{u}_j^2) =$$

$$-\tau_j S_j \pm \tau_i S_i + \rho_j h_j^t g \sin\beta - \frac{\partial}{\partial x}(h_j^t P_j) + P_{i,j}\frac{\partial h_j^t}{\partial x}$$

**Basic assumption: waves are long compared to the thickness of both layers**

The pressure at each phase varies only due to gravity :

$$\frac{\partial}{\partial x}(h_j^t P_j) = \rho_j h_j^t g \cos\beta \frac{\partial h_j^t}{\partial x} + \frac{\partial}{\partial x}(h_j P_{i,j});$$

$P_{i,1}=P_{i,2}$

### Closure relations

Wall and interfacial shear stresses $\tau_1, \tau_2, \tau_i \ = \tau_{1,2,i}(h_1^t, \bar{u}_1, \bar{u}_2, \partial h_1^t/\partial x)$

Shape factors $\gamma_1, \gamma_2$

$$\gamma_j = \frac{1}{h_j^t \bar{u}_j^2}\int_0^{h_j^t} u_j^2 \, dh_j^t;$$

# Two Fluid Model - Instability Condition

$$J_1 + J_2 + J_h \geq 1$$

*Brauner & Moalem (1991,1993)*

$$J_1 = \left(\frac{\rho_1}{\rho_1 - \rho_2}\right) \frac{U_{1S}^2}{Hg\cos\beta} \frac{1}{\tilde{h}^3} \left[ \left(\frac{C_w}{\bar{\bar{U}}_1} - 1\right)^2 + (\gamma_1 - 1)\left(1 - 2\frac{C_w}{\bar{\bar{U}}_1}\right) + \Delta\gamma_1 \right]$$

$$J_2 = \left(\frac{\rho_2}{\rho_1 - \rho_2}\right) \frac{U_{2S}{}^2}{Hg\cos\beta} \frac{1}{(1-\tilde{h})^3} \left[ \left(\frac{C_w}{\bar{\bar{U}}_2} - 1\right)^2 + (\gamma_2 - 1)\left(1 - 2\frac{C_w}{\bar{\bar{U}}_2}\right) + \Delta\gamma_2 \right]$$

**Terms due to phases' inertia, (K-H mechanism)**

$$J_h = C_h \frac{\rho(U_2 - U_1)^2}{(\rho_1 - \rho_2)Hg\cos\beta} \frac{1}{\tilde{h}(1-\tilde{h})}$$

**Wave-induced interfacial shear** $\propto \partial h_1^t / \partial x$
**("Sheltering" mechanism)**

$$\tau_{ih_x} = C_h \rho(\bar{U}_2 - \bar{U}_1)^2 \, \partial h_1^t / \partial x$$

**<u>Closures needed</u>:** $C_w(\tau_1, \tau_2, \tau_i)_{ss}$ and $\gamma_{1,2},\ \Delta\gamma_{1,2}$ (due to $\partial\gamma_{1,2} / \partial(h_1^t, \bar{u}_1, \bar{u}_2)$), $C_h$

Common assumptions :
$\gamma_{1,2} = 1,\ \Delta\gamma_{1,2} = 0,\ C_h = 0$

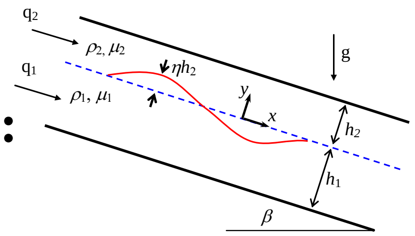
$C_w$ - Long wave **(kinematic wave)** **velocity-**affected **only** by the $\boldsymbol{\tau_{1,2,i}(h_1^t, \bar{u}_1, \bar{u}_2)}$ ***components in phase with the wave height*** – *i.e.*, closure relations used for obtaining the **steady-state** solution, $(\tau_1, \tau_2, \tau_i)_{ss}$

$$C_w = \left(\frac{dU_{1s}}{d\tilde{h}}\right)_{U_{1s}+U_{2s}=const,\ steady-state}$$

# Stability of Two-Layer Plane Poiseuille Flow

## Two-Plates (TP) Geometry

Follows the approach in the classical work of ***Yih (1967),*** *k*⟶*0***:**




# Destabilization Mechanisms:

**Shear flow instability-** Interaction of the fluids' flow with the channel walls.

- Encountered also in single-phase Poiseuille flow (Tollmien-Schlichting waves).
- Leads to transition to turbulent flow in either of the phases for sufficiently large Re.
- Associated with short waves instability.

**Interfacial instability-** Interaction between the fluids' flow at the interface

- Results from energy transfer from the main flow to interfacial disturbances.
- Instability is attributed to viscosity and/or density stratification (jump).
- Associated with (relatively) long waves instability.

**Kelvin-Helmholtz (K-H) mechanism???**

**Conclusion** (e.g., *Boomkamp & Miesen ,1996)*: All scheme for energy transfer to the waves, all related to viscous effects: "Including **viscous effects**, however small, into the stability problem **rules out** the possibility of the essentially inviscid **K-H** instability"

# Stability of Two-Layer Plane Poiseuille Flow

*Barmak et al., Phys. Fluids (2016a,b)*

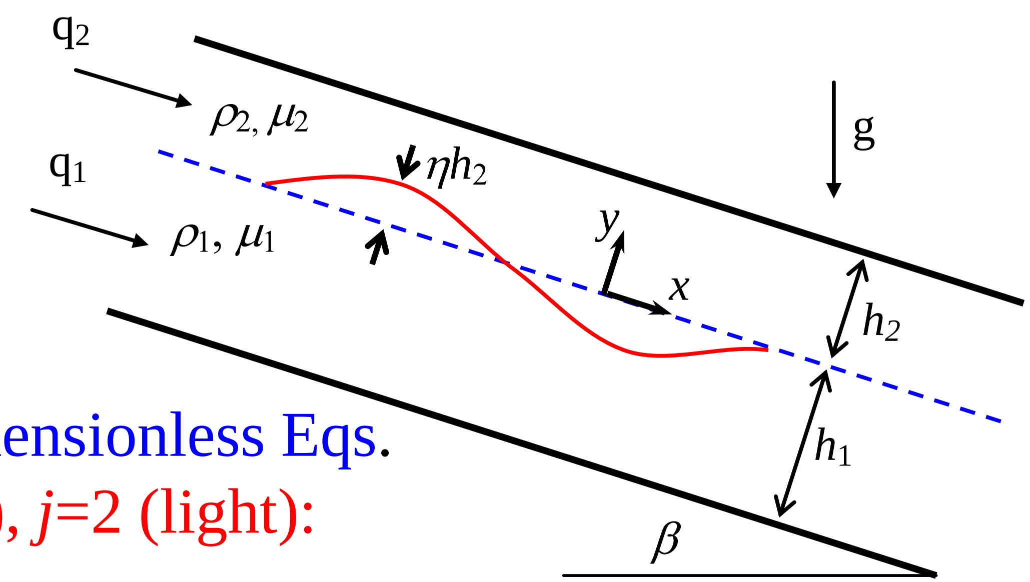
## Objectives

- Determine the range of flow parameters for which smooth-stratified flow can be stable, while considering all perturbations modes (all $k$).
- Identify the most dangerous perturbation mode in various systems, horizontal and inclined, and where they are initiated in the flow.
- Present the results in the form of stability boundaries on flow pattern maps for the sake of further physical interpretations.
- Identify systems & conditions where long waves are the most dangerous perturbation, for which exact analytical solution is available.
- Examine the possibility of using the the Two-Fluid (TF) model for reproducing the exact long-wave stability boundary, thereby identifying the closures needed & the dominant destabilizing mechanism.

# Exact Analysis: Two-Layer Plane Poiseuille Flow:

## 2D transient laminar two-layer flow between Two-Plates (TP)

$$\left(u_j, v_j\right) = \left(\hat{u}_j, \hat{v}_j\right)/u_i;$$
$$\left(x, y\right) = \left(\hat{x}, \hat{y}\right)/h_2;$$
$$p_j = \hat{p}_j \big/ \rho_2 u_i^2; \; t = \hat{t} u_i \big/ h_2$$




Continuity & momentum dimensionless Eqs.

*j*=1 (heavy), *j*=2 (light):

$$\frac{\partial u_j}{\partial x} + \frac{\partial v_j}{\partial y} = 0$$

$$\frac{\partial u_j}{\partial t} + u_j \frac{\partial u_j}{\partial x} + v_j \frac{\partial u_j}{\partial y} = -\frac{\rho_1}{r \rho_j} \frac{\partial p_j}{\partial x} + \frac{1}{\mathrm{Re}_2} \frac{\nu_j}{\nu_1} \frac{m}{r} \left( \frac{\partial^2 u_j}{\partial x^2} + \frac{\partial^2 u_j}{\partial y^2} \right) + \frac{\sin \beta}{\mathrm{Fr}_2}$$

$$\frac{\partial v_j}{\partial t} + u_j \frac{\partial v_j}{\partial x} + v_j \frac{\partial v_j}{\partial y} = -\frac{\rho_1}{r \rho_j} \frac{\partial p_j}{\partial y} + \frac{1}{\mathrm{Re}_2} \frac{\nu_j}{\nu_2} \frac{m}{r} \left( \frac{\partial^2 v_j}{\partial x^2} + \frac{\partial^2 v_j}{\partial y^2} \right) - \frac{\cos \beta}{\mathrm{Fr}_2}$$

$$\mathrm{Re}_2 = \frac{u_i h_2}{\nu_2}; \quad m = \mu_1 / \mu_2$$
$$\mathrm{Fr}_2 = \frac{u_i^2}{g h_2}; \quad r = \rho_1 / \rho_2$$

**Boundary conditions**:

Channel walls - no-slip. Interface - continuity of the velocity components, balance of the stress components & kinematic condition.

# Temporal Stability Analysis

Governing differential system formulated as an eigenvalue problem

$$u_j = U_j + \breve{u}_j, \quad v_j = \breve{v}_j, \quad p_j = P_j + \breve{p}_j; \qquad \breve{u}_j = \frac{\partial \Phi_j}{\partial y}, \quad \breve{v}_j = -\frac{\partial \Phi_j}{\partial x}$$

$\eta = \breve{\eta}$ - disturbance of the interface height ; $\Phi$ - disturbance stream function

$$\begin{pmatrix} \Phi \\ \breve{p}_j \\ \breve{\eta} \end{pmatrix} = \begin{pmatrix} \phi_j(y) \\ f_j(y) \\ H \end{pmatrix} e^{(ikx + \lambda t)}$$

$$\lambda = \lambda_R + i\lambda_I = -ik(c_R + ic_I)$$

Neutral stability: $\lambda_R = 0$

$$k = \frac{2\pi h_2}{L_{wave}}$$

**Orr-Sommerfeld equations-** solved numerically by the Chebyshev Collocation Spectral Method

$$\lambda D_j \phi_j = \left[ ik\left(-U_j D_j + U_j''\right) + \frac{1}{\mathrm{Re}_j} D_j^2 \right] \phi_j \qquad (j = 1, 2) \qquad + \quad \text{Boundary conditions}$$

with $D_1 = \frac{\phi''}{n^2} - k^2\phi,\ D_1^2 = \frac{\phi^{IV}}{n^4} - 2k^2\frac{\phi''}{n^2} + k^4\phi;\ D_2 = \phi'' - k^2\phi,\ D_2^2 = \phi^{IV} - 2k^2\phi'' + k^4\phi.$

Given $k$, $Y, q, m = \mu_1/\mu_2,\ r = \rho_1/\rho_2,\ \mathrm{Re}_2,\ \mathrm{Fr}_2,\ \mathrm{We}_2 = \rho_2 h_2 u_i^2/\sigma$ → $\phi_{1,2}(y), \lambda$

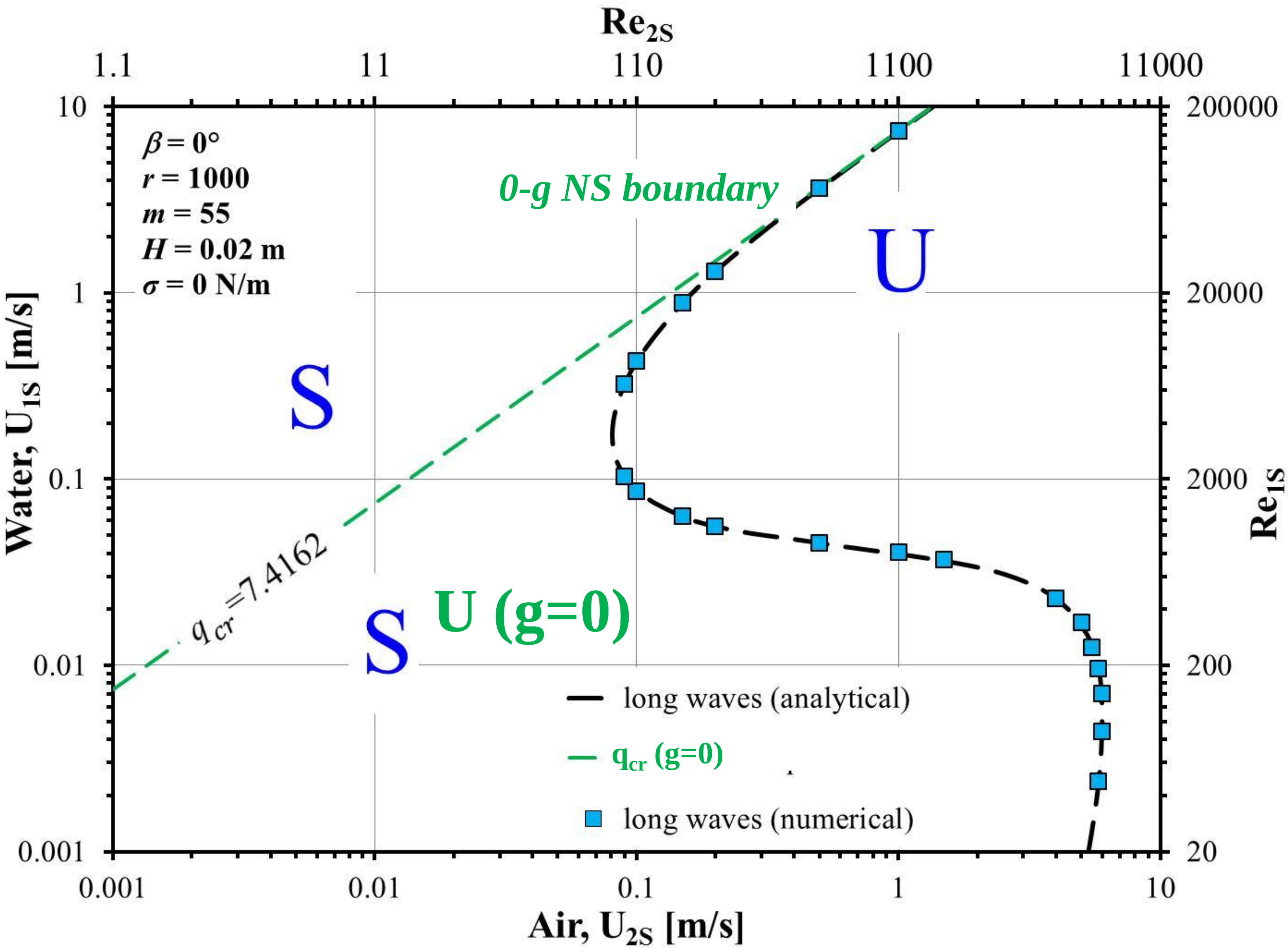
Stable stratified flow corresponds to conditions for which disturbances of all $k$`s are damped

Critical disturbance: the particular $k$ which is responsible for triggering the instability

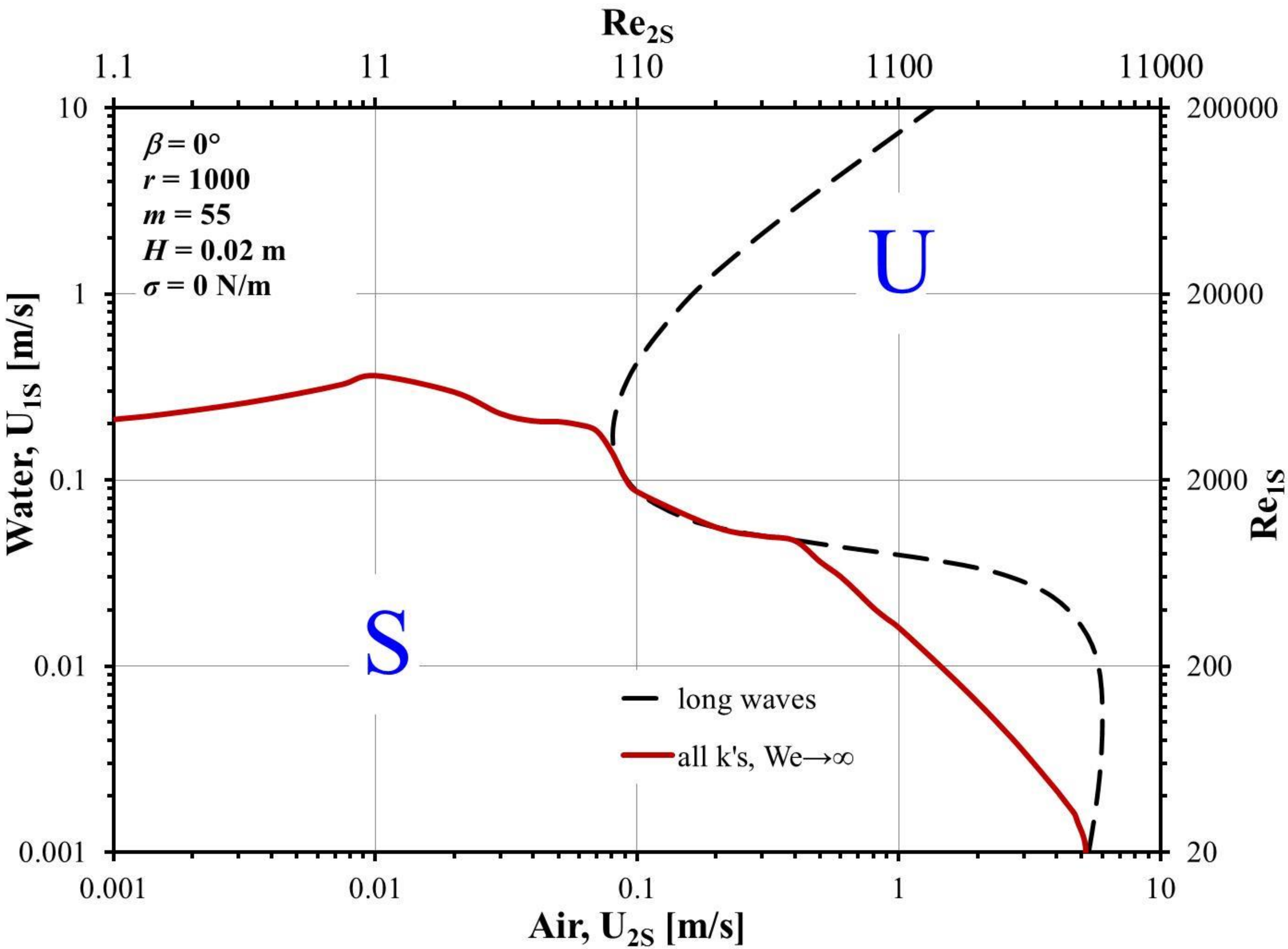
# Case Study- Horizontal air-water system, *H*=0.02m

## Stability map – Long waves, *k*→*0*

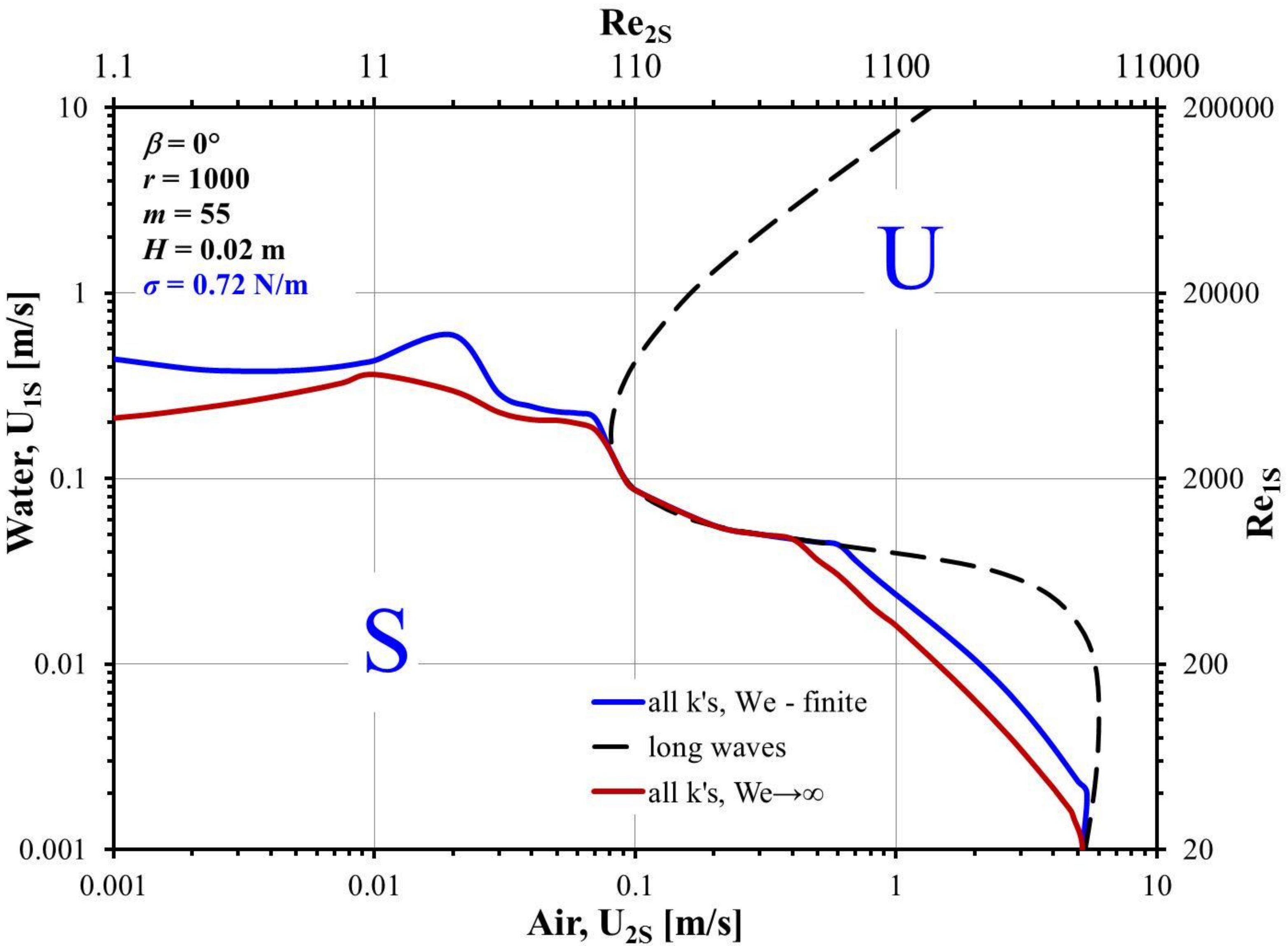
**Long wave (LW) –analytical asymptotic solution** (*Kushnir et al., IJMF, 2014*)



# Horizontal air-water system : Stability map (all k's) We→∞ (σ=0)

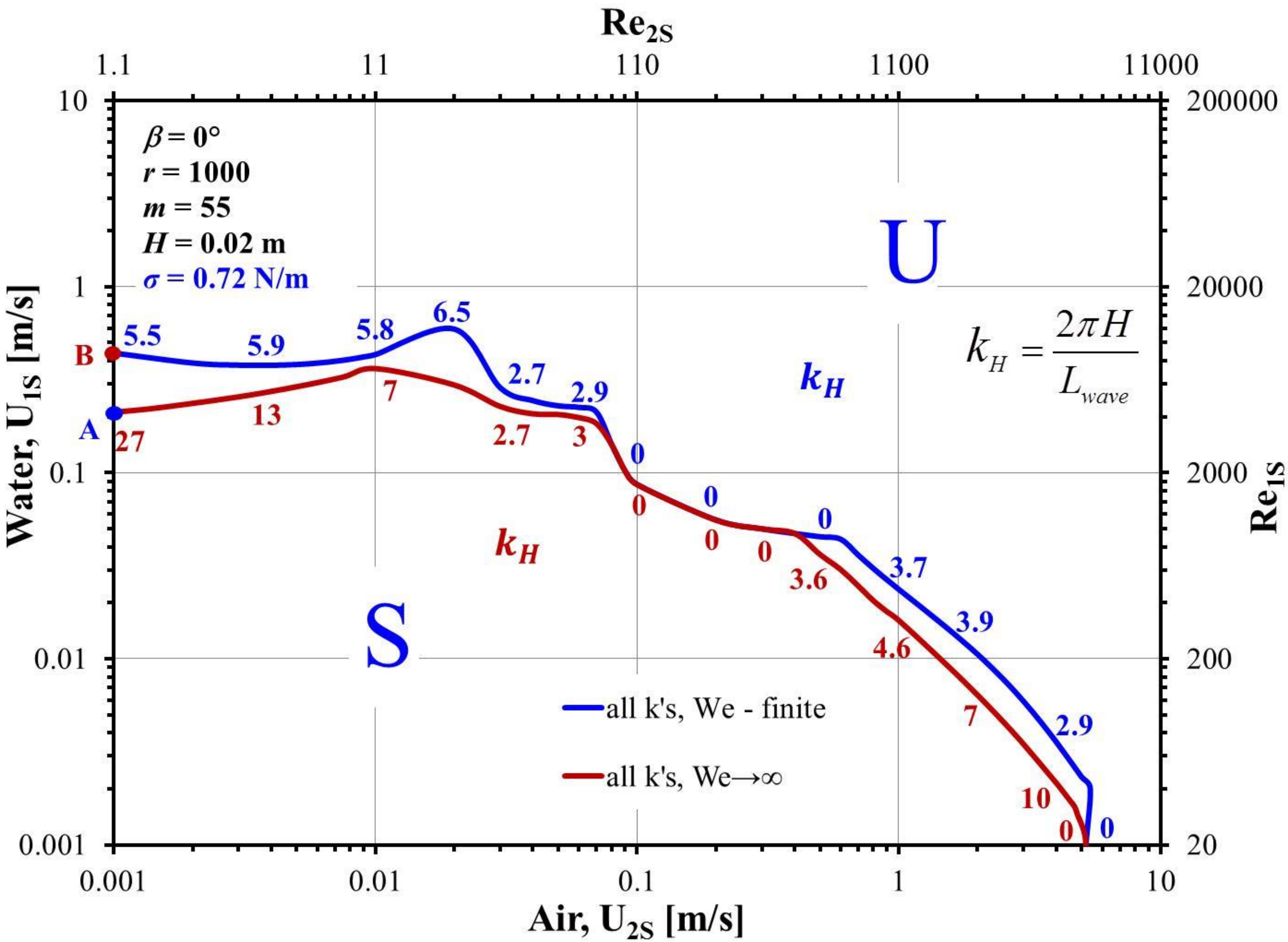


# Horizontal air-water system : Stability map (all k's)
# Surface tension included



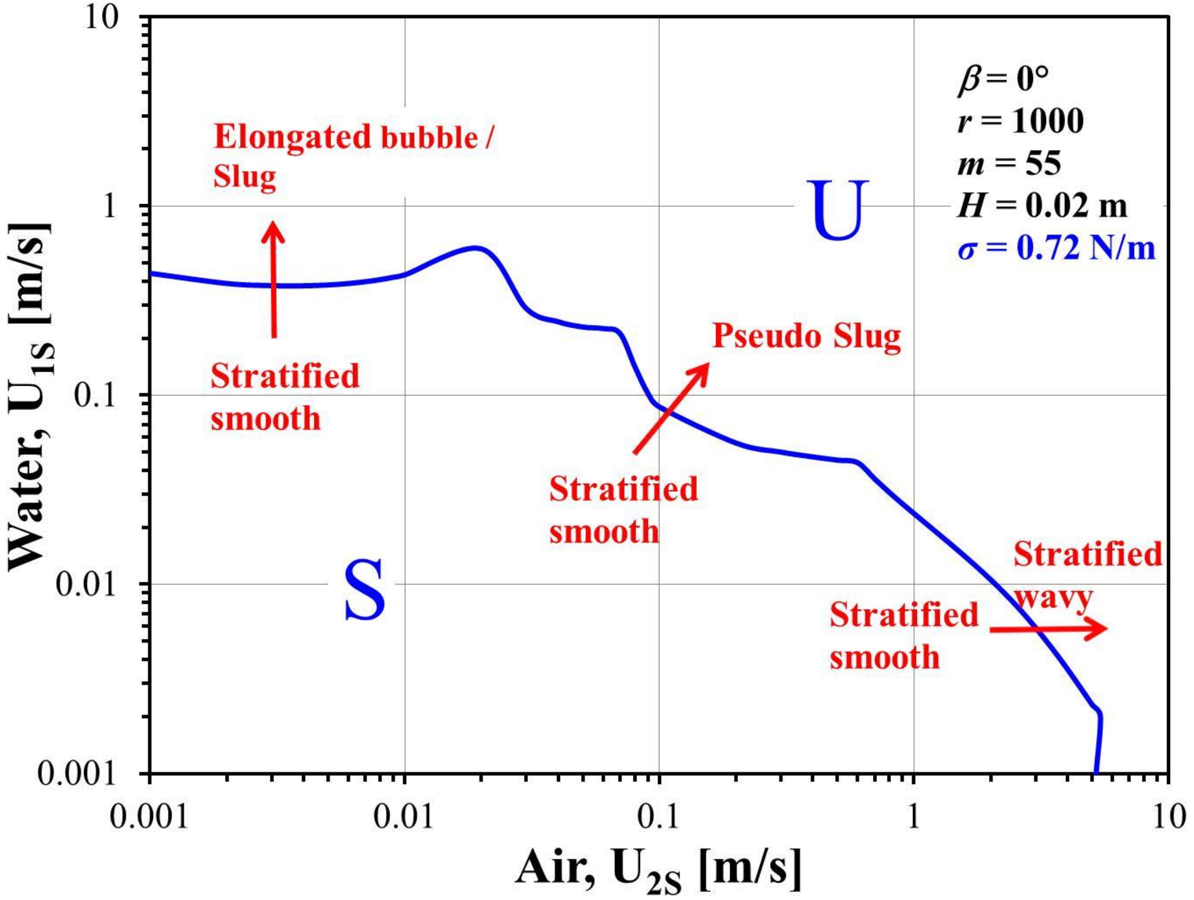
# Horizontal air-water system: Stability map (all k's)

## Effect of surface tension on the critical disturbance- shift to lower *k*

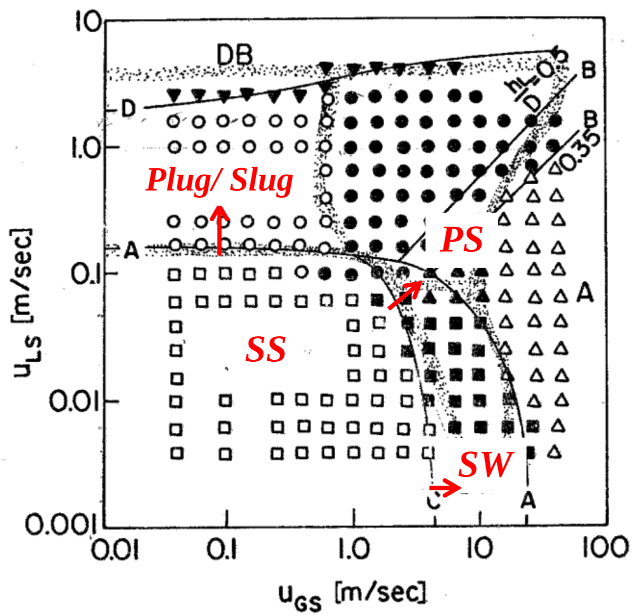


# Horizontal air-water system

## Flow pattern transitions across stability boundary

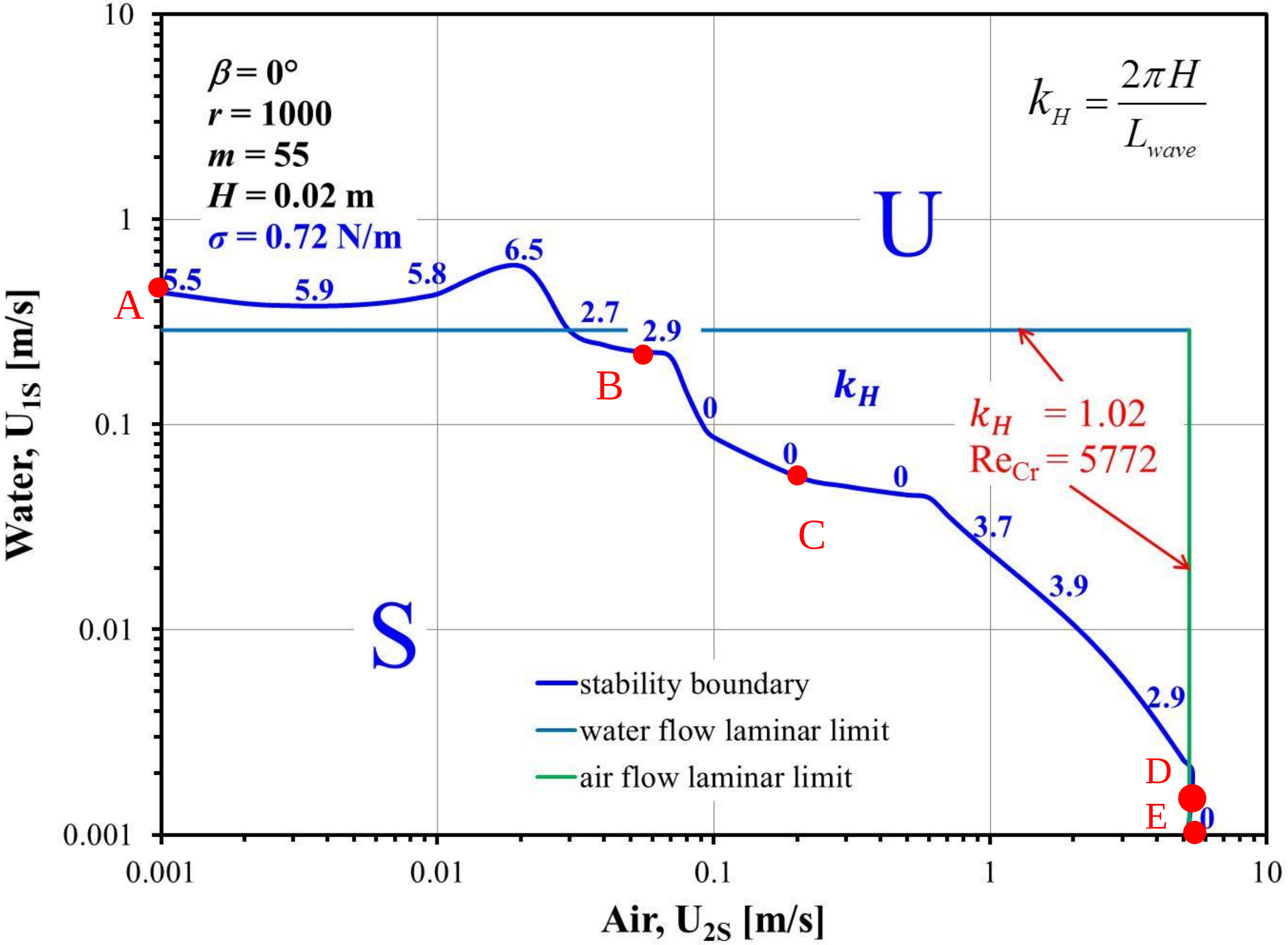



Experimental data D=2.54cm
*Barnea et al. (1980)*



# Horizontal air-water system (all k's)

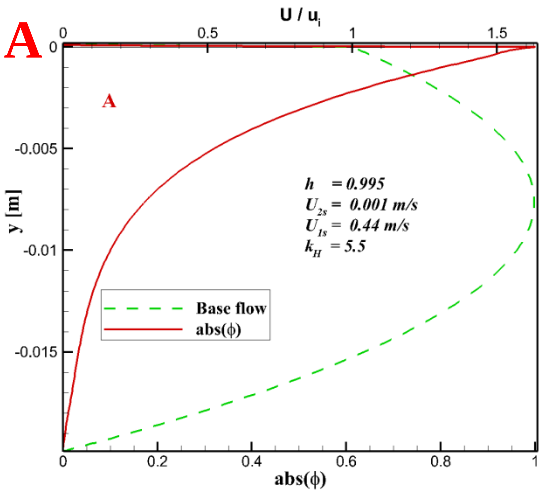
## Single phase flow stability limits for Air and Water (shear instability)

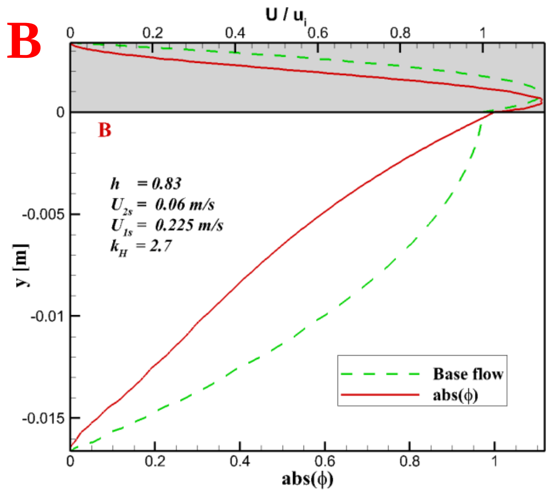


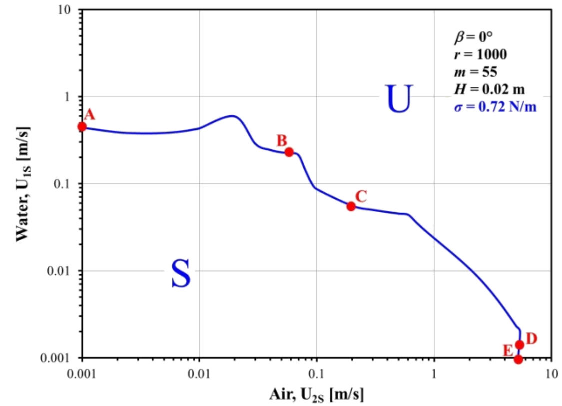
# Amplitude of the stream function perturbations (Eigenfunctions)

## Along the neutral stability curve (sample points)

### Short wave, k >1; Interfacial mode



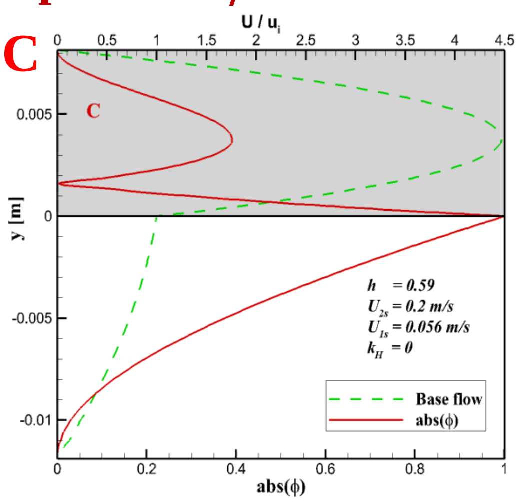

Perturbation Stream Function Amplitude - $\phi$

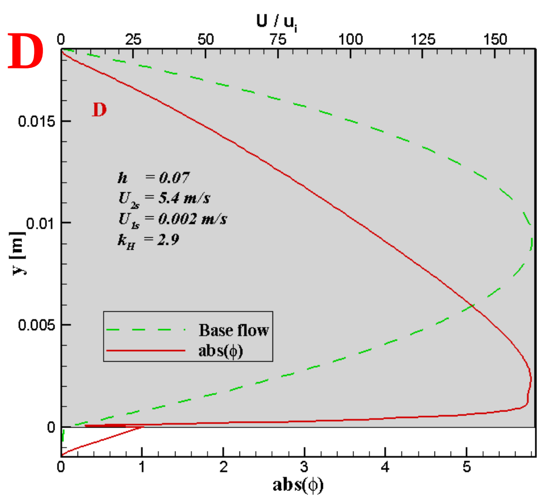
Velocity Profile of the base flow – $U/u_i$

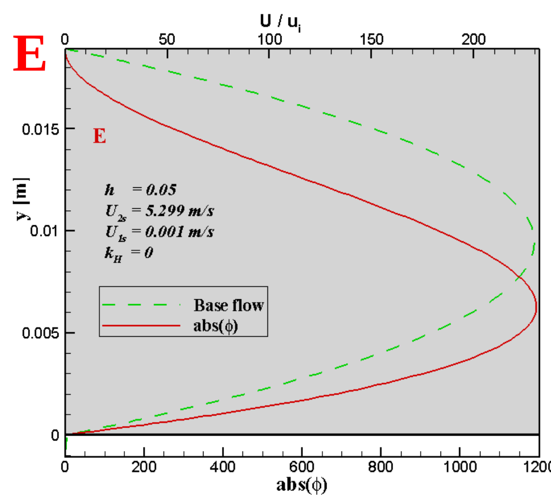
Short wave, k >1; Shear mode

Long wave, k →0 Shear mode

Long wave, k →0; Interfacial mode



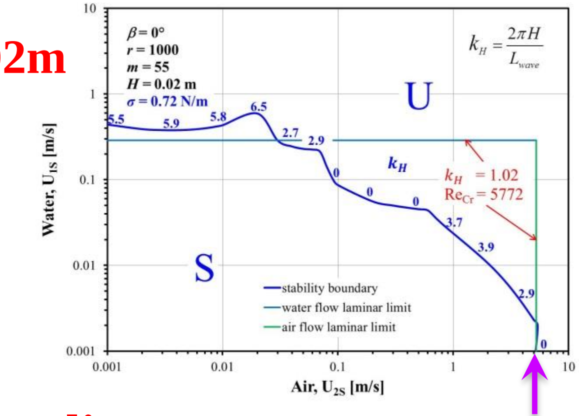



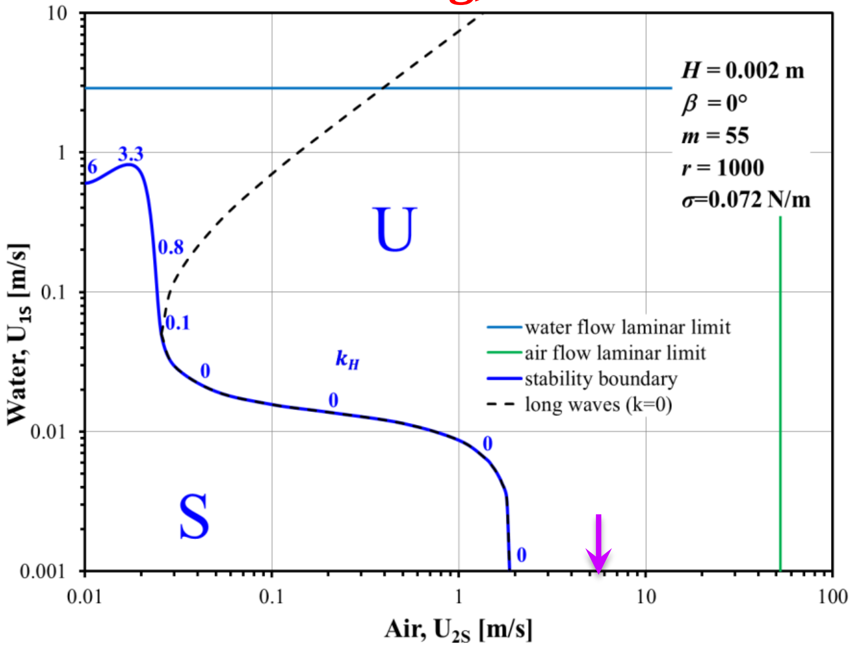
# Horizontal air-water system

# Channel size effect

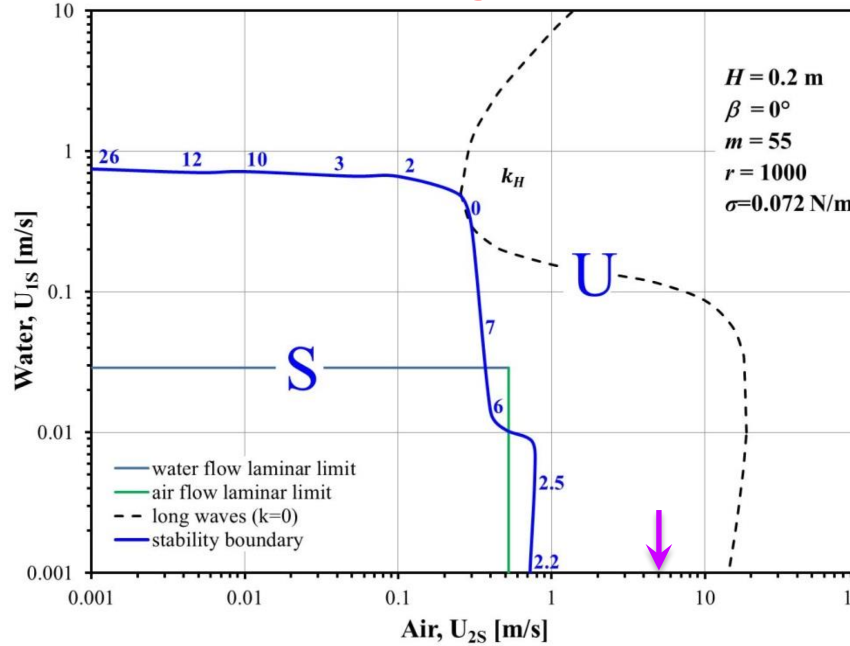
*H*=0.02m




## Down-scaling, *H*=0.002m




## Up-scaling, *H*=0.2m




**Critical $U_{2s}$ - Scaled by $(Fr_2)_{crit}$ (H< ≈2cm)**

$$(\mathrm{Fr}_2)_{crit} = \left(\frac{\rho_2}{\Delta\rho g H}\right)^{0.5} (U_{2s})_{crit}\,; \quad H\downarrow (U_{2s})_{crit}\downarrow$$

**Critical $U_{2s}$ - Scaled by $(Re_2)_{crit}$ (H> ≈2cm)**

$$(\mathrm{Re}_2)_{crit} = \frac{\rho_2 H (U_{2s})_{crit}}{\mu_2}\,; \quad H\uparrow (U_{2s})_{crit}\downarrow$$

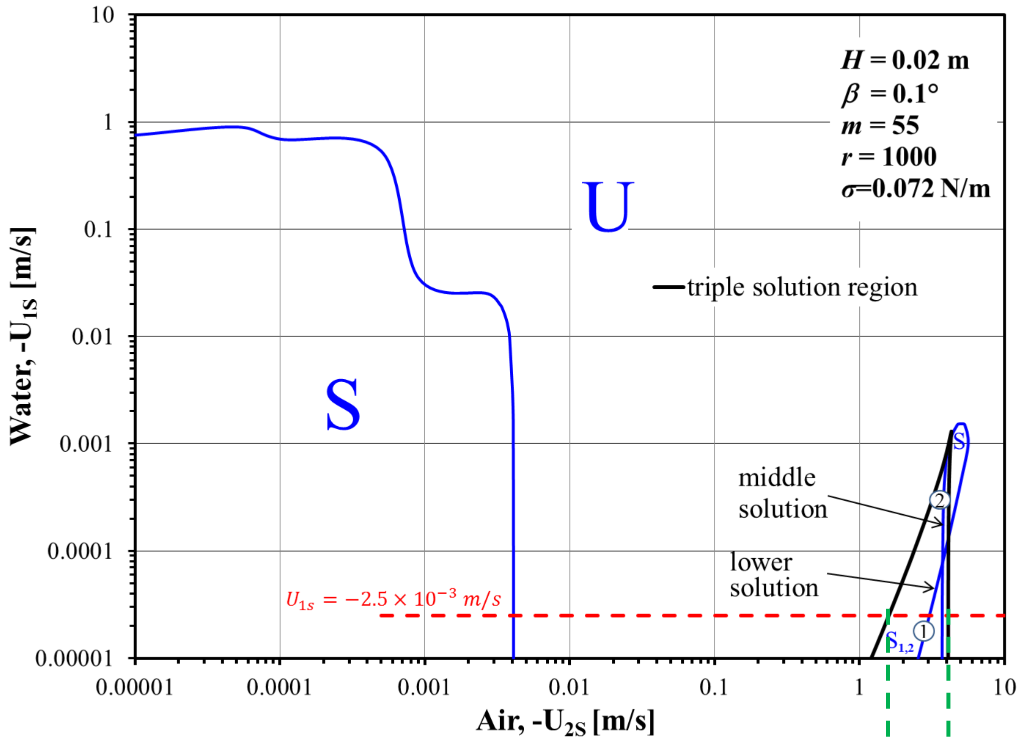
**Air superficial velocity for SS/SW transition is maximal in *H* ≈0.024m, ≈ 5.75m/s**

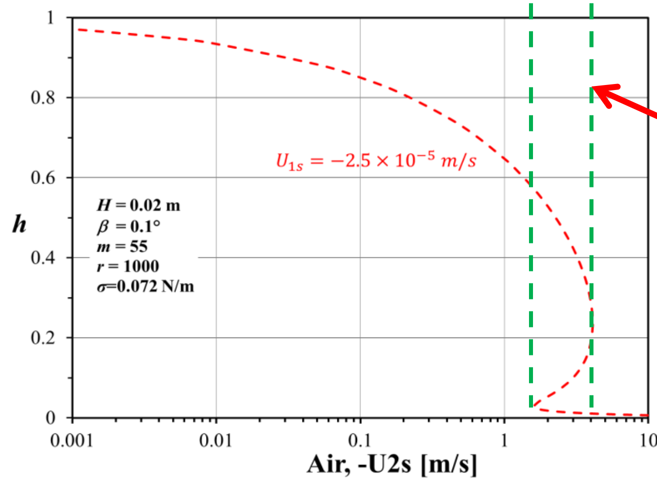
# Stability in Inclined Flows

## Some case studies

# Upward inclined air-water flow – Stability map

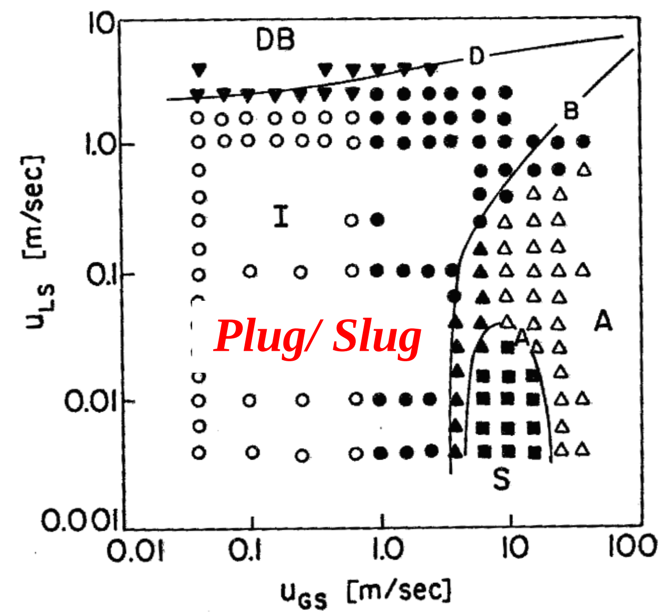
## $\beta$=0.1°, $H$=0.02m




Experimental data D=2.5cm, $\beta$=0.25°

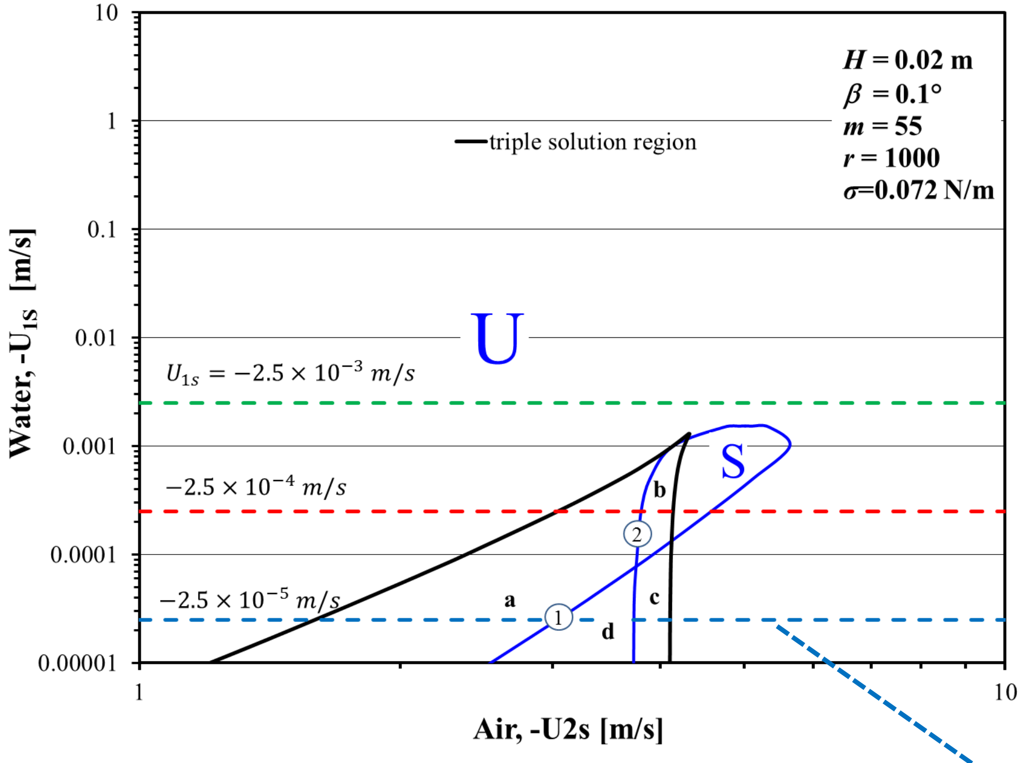
*Barnea et al. (1980)*




Triple solution region

# Upward inclined air-water flow – Stability map

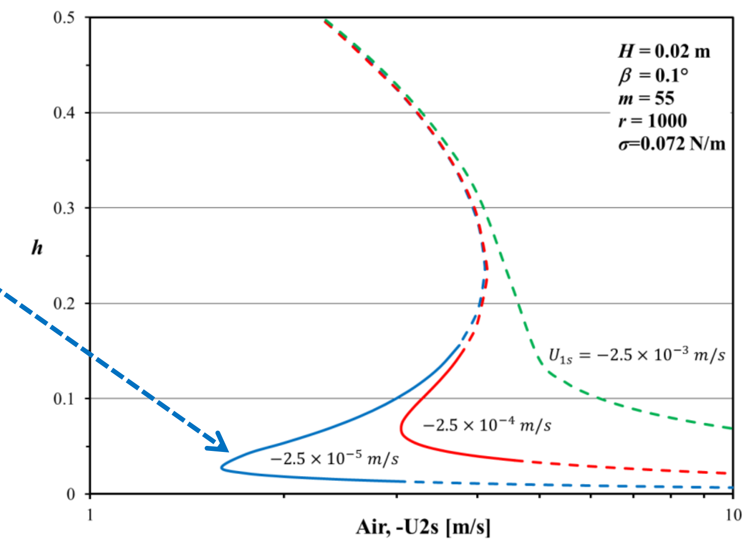
## In the Triple solution region




**a** - lower and middle solutions are stable
**b** - only the lower solution is stable

**c** – all three solutions are unstable
**d** – only the middle solution is stable

Bold curves – stable conditions
Dashed curves – unstable




**The upper solution is always unstable!**

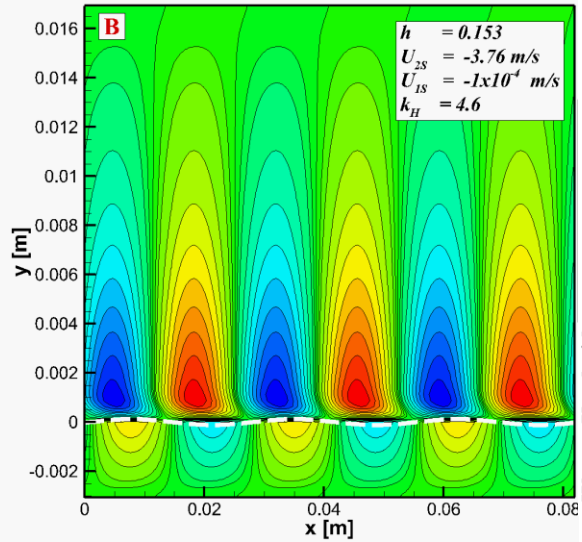
Long-wave analysis ($k \to 0$):
middle solution is always stable.
Upper solution is stable in part of the 3-s region.

# Upward inclined air-water flow

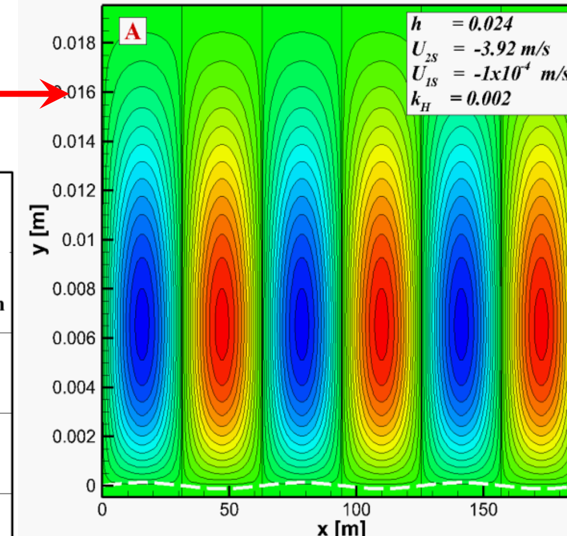
## Perturbed flow patterns at neutral stability

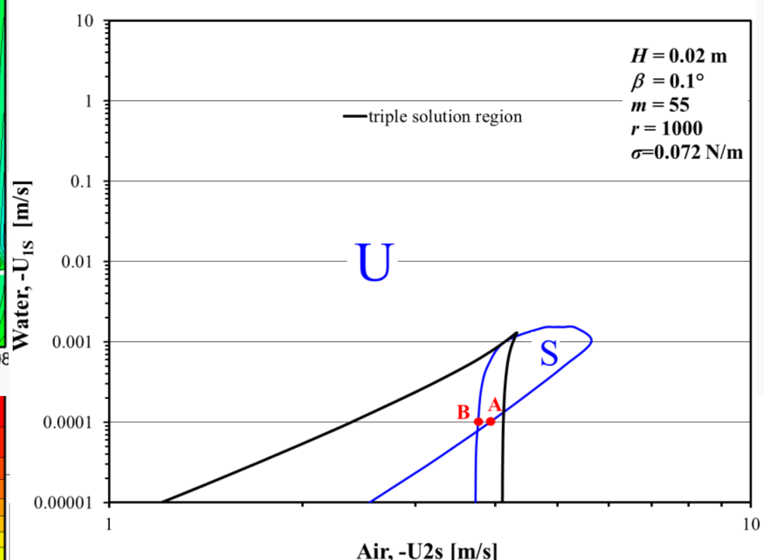
**B. Middle solution**
**Short wave, k =4.6**
**Interfacial mode**

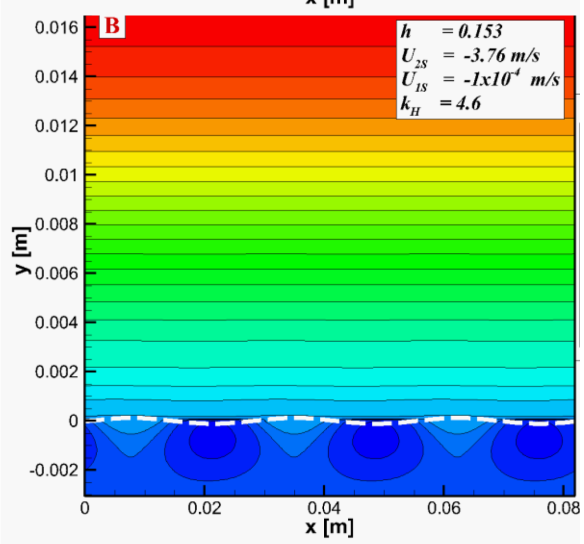
**A. Lower solution**
**Long wave, k →0**
**Shear mode**



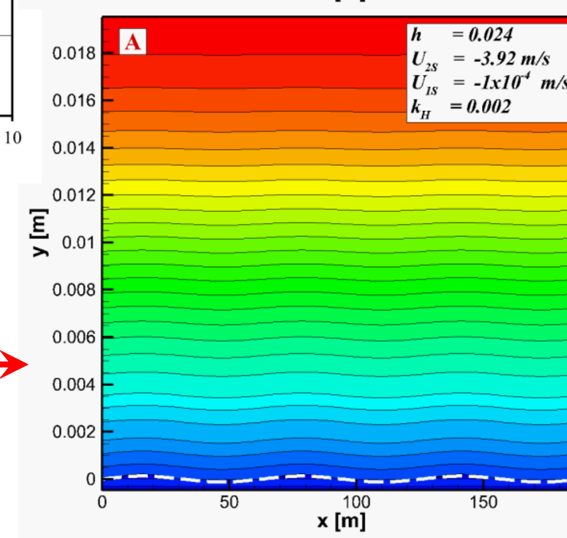

**Disturbance streamlines**



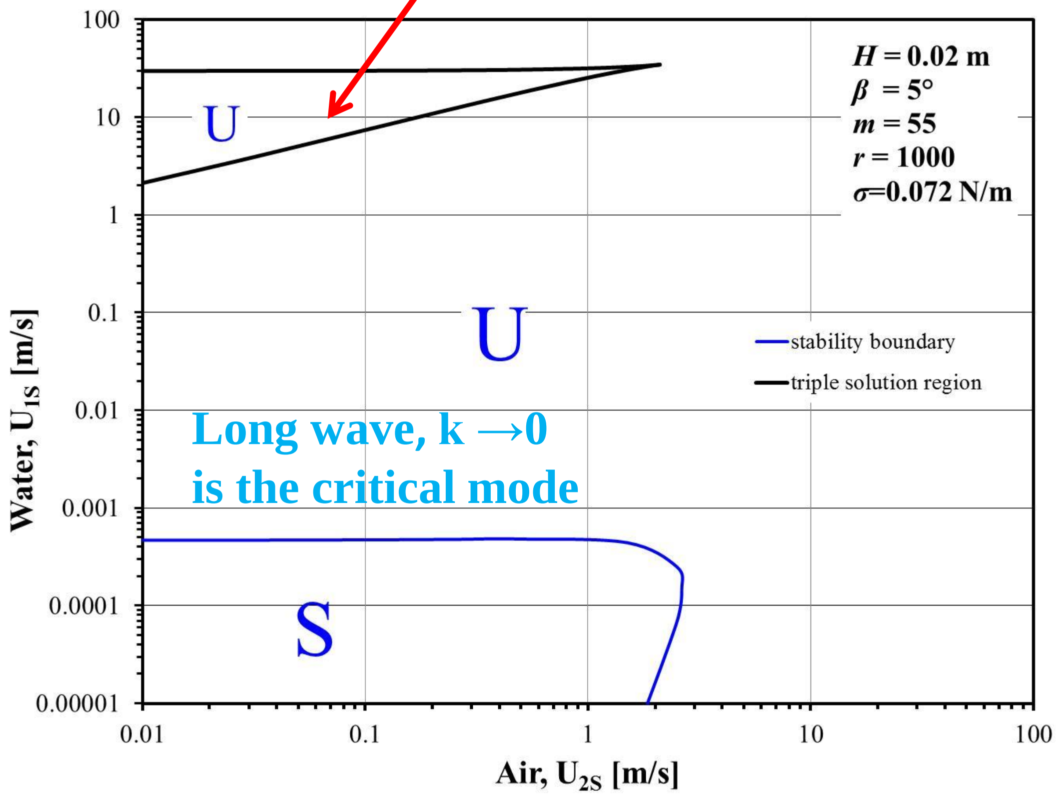




**Perturbed flow streamlines**
**Disturbance +main flow**

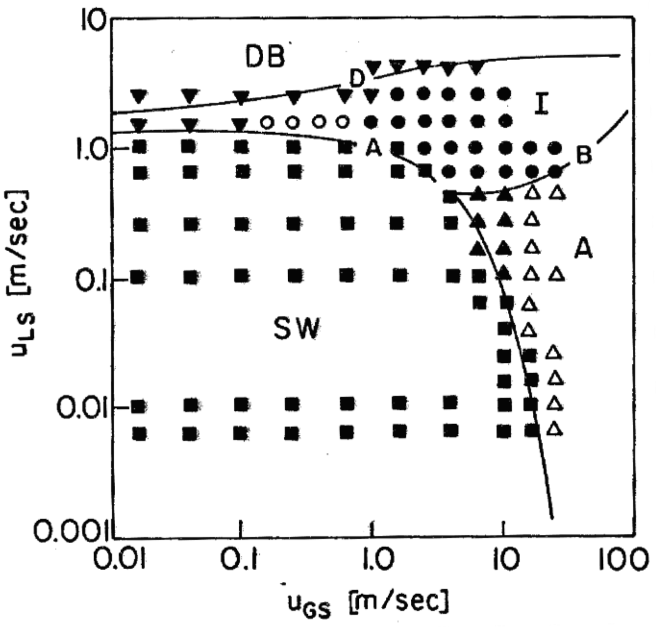


# Downward inclined air-water flow Stability map
## $\beta$=5°, $H$=0.02m

**Triple solution region, unstable!**




Experimental data D=2.54 cm
*Barnea et al. (1982)*




Long-wave analysis ($k \to 0$) predicts multiple stable solutions in the triple solution region.

## **Observation:**

There are systems and conditions where long waves are the most ‘dangerous’ mode for triggering instability.

Long waves is the underlying assumption of the TF model

Can the Long Wave (LW) stability boundary be reproduced by the Two-Fluid Model ?

Are the required closure relations available??

# Two Fluid Model - Neutral stability condition

*Kushnir et al., IJMF, (2017)*

$$J_1 + J_2 + J_h = 1$$

$$J_1 = \left(\frac{\rho_1}{\rho_1 - \rho_2}\right) \frac{U_{1S}^2}{Hg\cos\beta} \frac{1}{\tilde{h}^3} \left[ \left(\frac{C_w}{\overline{\overline{U}}_1} - 1\right)^2 + (\gamma_1 - 1)\left(1 - 2\frac{C_w}{\overline{\overline{U}}_1}\right) + \Delta\gamma_1 \right]$$

$$J_2 = \left(\frac{\rho_2}{\rho_1 - \rho_2}\right) \frac{U_{2S}{}^2}{Hg\cos\beta} \frac{1}{\left(1 - \tilde{h}\right)^3} \left[ \left(\frac{C_w}{\overline{\overline{U}}_2} - 1\right)^2 + (\gamma_2 - 1)\left(1 - 2\frac{C_w}{\overline{\overline{U}}_2}\right) + \Delta\gamma_2 \right]$$

**Terms due to phases' inertia, (K-H mechanism)**

$$J_h = \frac{K_i}{(\rho_1 - \rho_2)Hg\cos\beta} \frac{(1-\tilde{h})K_1/K_i - \tilde{h}K_2/K_i - 1}{\tilde{h}(1-\tilde{h})}$$

**Wave-induced shear stresses**

$$\tau_{1,2,i} = K_{1,2,i}\,\partial h_1^t / \partial x$$

**("Sheltering" mechanism)**

**Known from the SS solution:**

$C_w(\tilde{h}, q, m, Y)$ $\quad \gamma_{1,2}(\tilde{h}, q, m, Y)$ $\quad \Delta\gamma_{1,2}(\tilde{h}, q, m, Y)$

**Wave velocity** **Shape factors & derivatives**

$K_{1,2,i}$ - The exact LW analytical solution was used to obtain the wave induced interfacial and wall shear stresses components in phase with the wave slope (*Kushnir et al., 2014*):

$$\tau_{1,2,i} = K_i\,\partial h_1^t/\partial x,\ \ K_2\,\partial h_1^t/\partial x,\ \ K_i\,\partial h_1^t/\partial x \quad \xrightarrow[\textbf{Closures}]{\textbf{Analytical}} \quad K_{1,2,i}(q, m, r, Y, \mathrm{Re}_2, \mathrm{Fr}_2)$$

# Modeling of the 'Sheltering' Term, $J_h$ *Kushnir et al., IJMF, (2017)*

$$J_h = \mathbf{C_h} \frac{\mathrm{Fr}_{2s}^2}{\tilde{h}(1-\tilde{h})^3 \cos(\beta)} \left(1 - \frac{\bar{U}_1}{\bar{U}_2}\right) \text{ for } \bar{U}_2 \geq \bar{U}_1 \quad (m \geq 1) \qquad \mathrm{Fr}_{2s}^2 = \frac{U_{2s}^2}{(r-1)\, gH}$$

$$J_h = \mathbf{C_h} \frac{\mathrm{Fr}_{1s}^2}{\tilde{h}^3(1-\tilde{h}) \cos(\beta)} \left(1 - \frac{\bar{U}_2}{\bar{U}_1}\right) \text{ for } \bar{U}_1 > \bar{U}_2 \quad (m < 1) \qquad \mathrm{Fr}_{1s}^2 = \frac{U_{1s}^2\, r}{(r-1)\, gH}$$

## $\mathbf{C_h}$-an apparent sheltering coefficient, combines the effects of $K_i, K_1, K_2$

**Asymptotic $C_h$ values**

| $C_h$ $\tilde{h}\to 0$ | $m$ |
|---|---|
| **0.00255** | **1.01** |
| **0.01682** | **1.07** |
| **0.08571** | **1.5** |
| **0.12857** | **2** |
| **0.20571** | **5** |
| **0.24429** | **20** |
| **Air-water** **0.25247** | **55** |
| **0.25709** | **5000** |
| **0.25714** | **50000** |
| **0.25714** | **500000** |

***Tabulated values** of* **$C_h$ ($m$, $r$, $\tilde{h}$), m>1 & m<1** *Kushnir et al., IJMF (2017)*

***s*** -"Sheltering coefficient" of Jeffreys vs. $\mathbf{C_h}$

$$s = \frac{2C_h\mu_1}{\rho_1 U_{1s} H} = \frac{4C_h}{\mathrm{Re}_{1s}} \equiv \frac{4\mathbf{C_h}}{\mathrm{Re}_{Ls}}$$

$$\tilde{h}\to 0,\ \mathrm{Re}_{Ls}\to 0,\ s\to\infty$$

***No asymptotic value for s !***

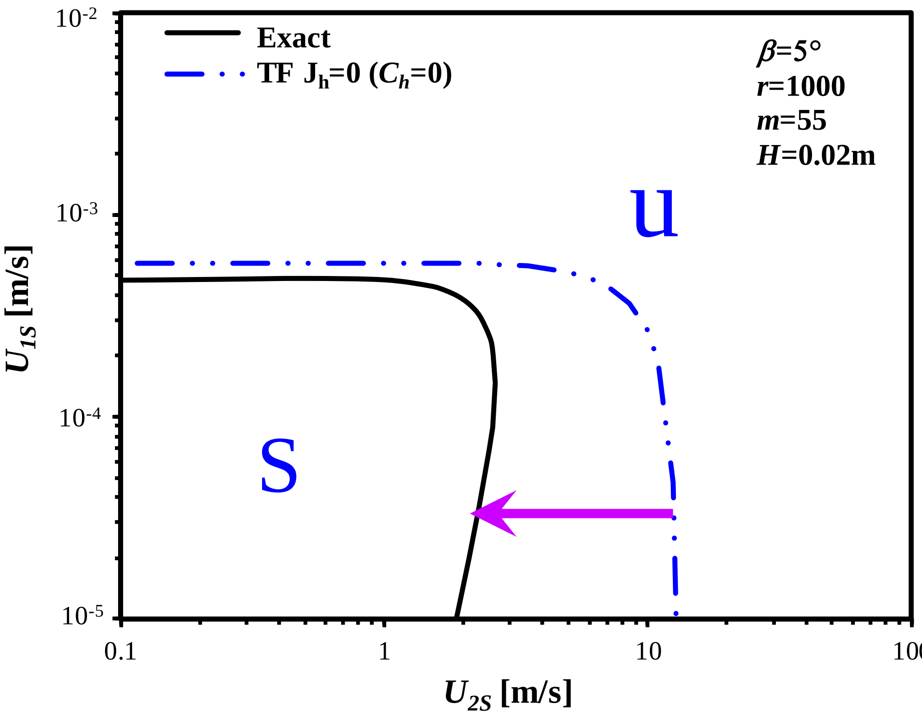
# The impact of $J_h$ on the L-W stability boundary

**Air-water system, $H$=0.02m**

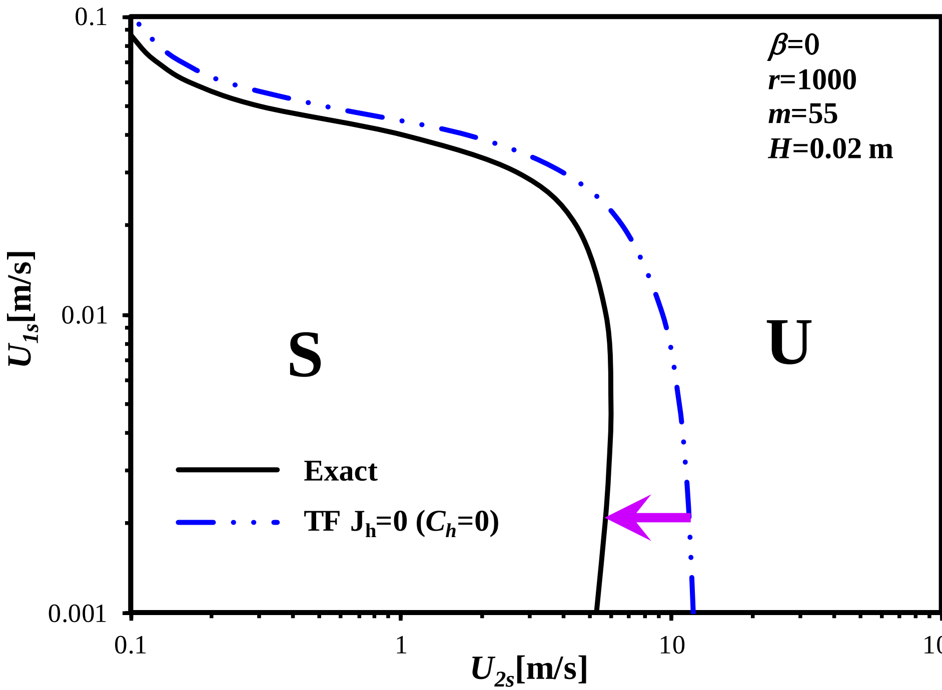
**TF neutral stability curves $J_h=0$ ($C_h=0$)**

**TF** with $J_h$ ($C_h \neq 0$) $\equiv$ **Exact long wave neutral stability curve**

## Concurrent downward flow




## Horizontal flow




Using the **Sheltering term** (with the closures for the $C_h$ (or *Ks'* ) obtained from the L-W analysis) the **TF model** reproduces the **exact L-W stability boundary**

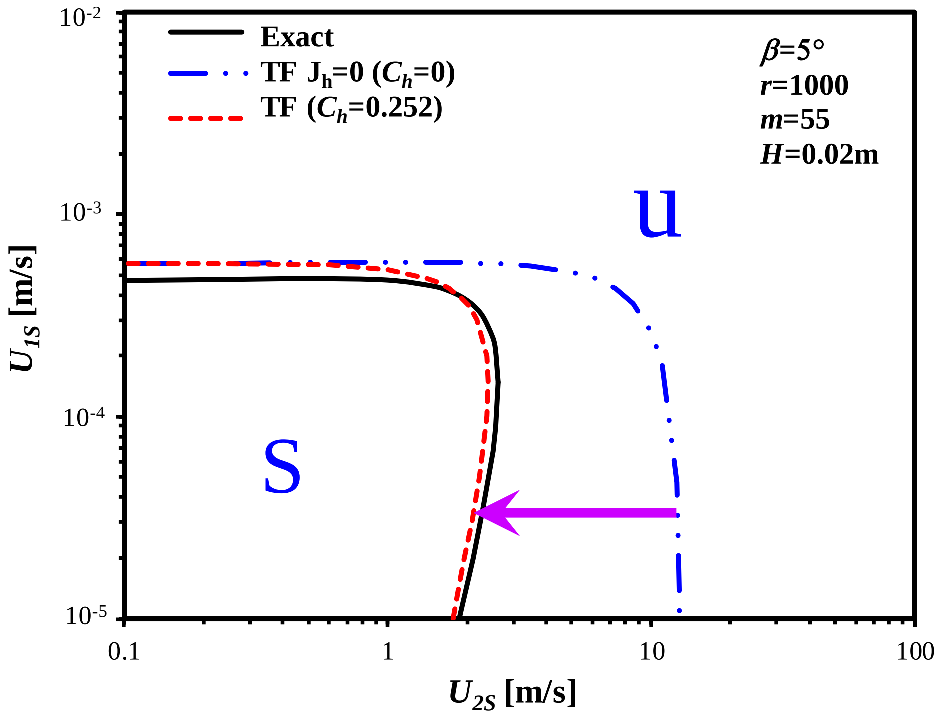
# The impact of $J_h$ on the L-W stability boundary

**Air-water system, $H$=0.02m**

**TF neutral stability curves $J_h=0$ ($C_h$=0)**

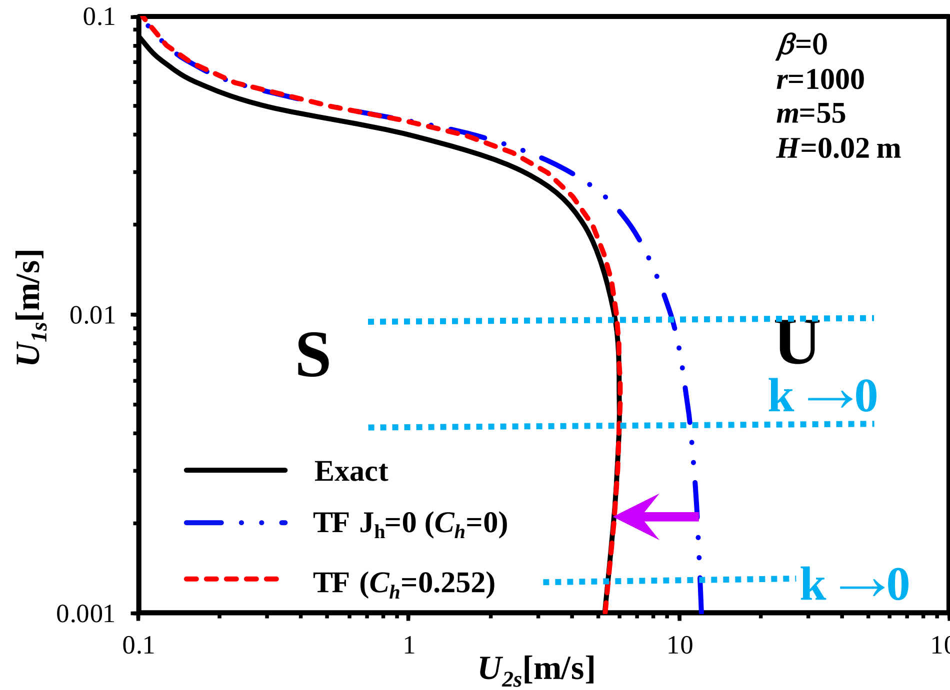
**TF with $J_h$ ($C_h$=0.252) ≈ Exact long wave neutral stability curve**

## Concurrent downward flow




**Long wave, k →0 is the critical mode all along the stability boundary**

## Horizontal flow

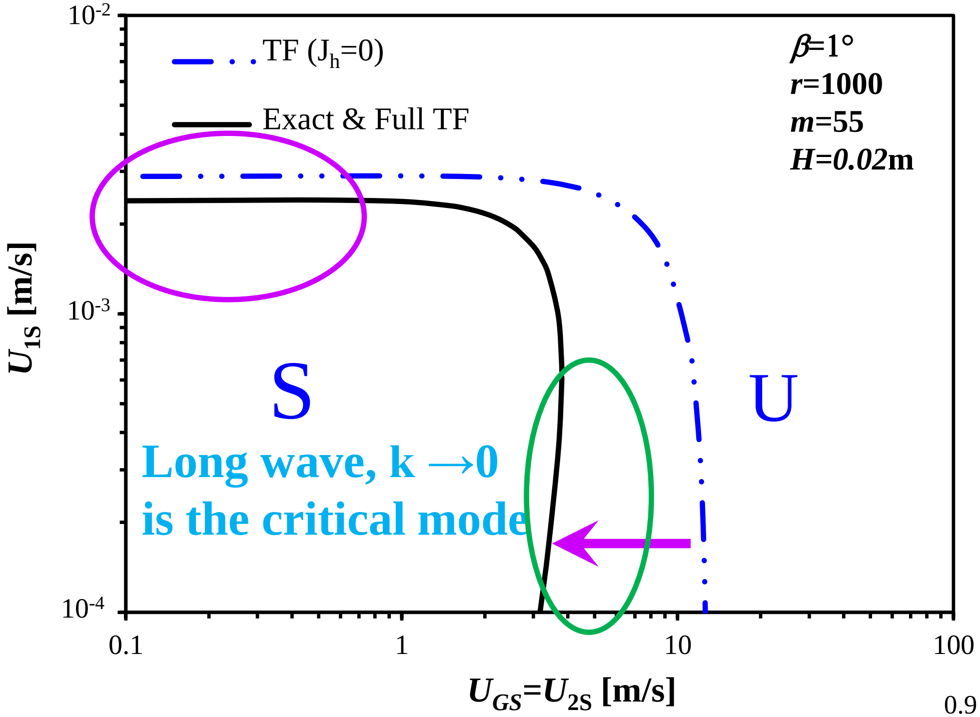



**Long wave, k →0 is the critical mode only in parts of the stability boundary**

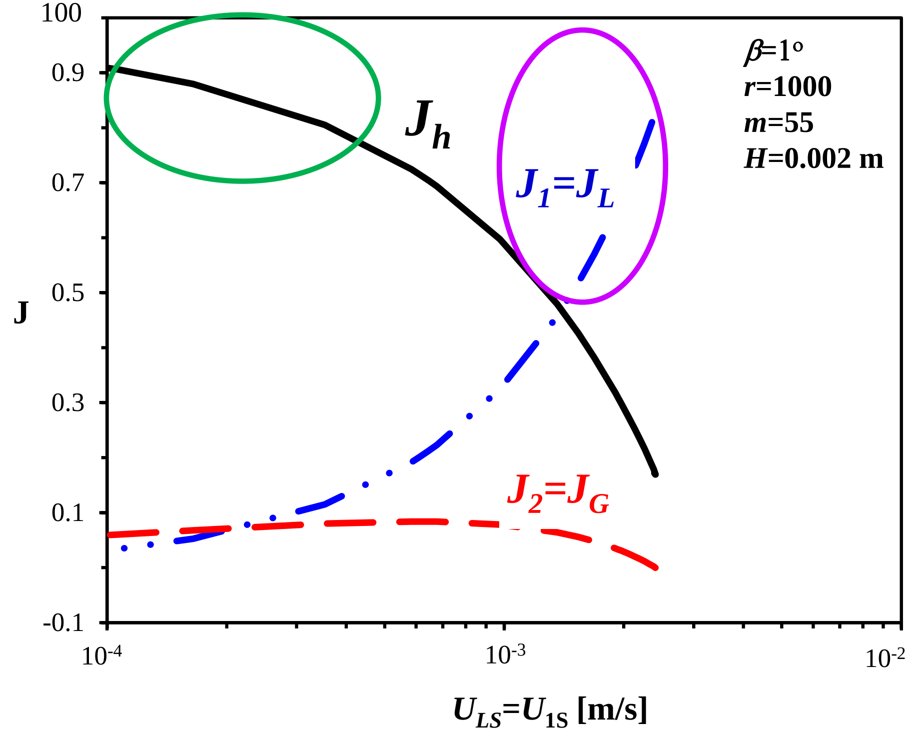
# Exact LW & TF model

## Co-current Down, $\beta$=1°

Air-water system H=0.02m




## Role of $J_1$, $J_2$, $J_h$




**$J_h$ ('Sheltering') is dominant at low $U_{Ls}$**

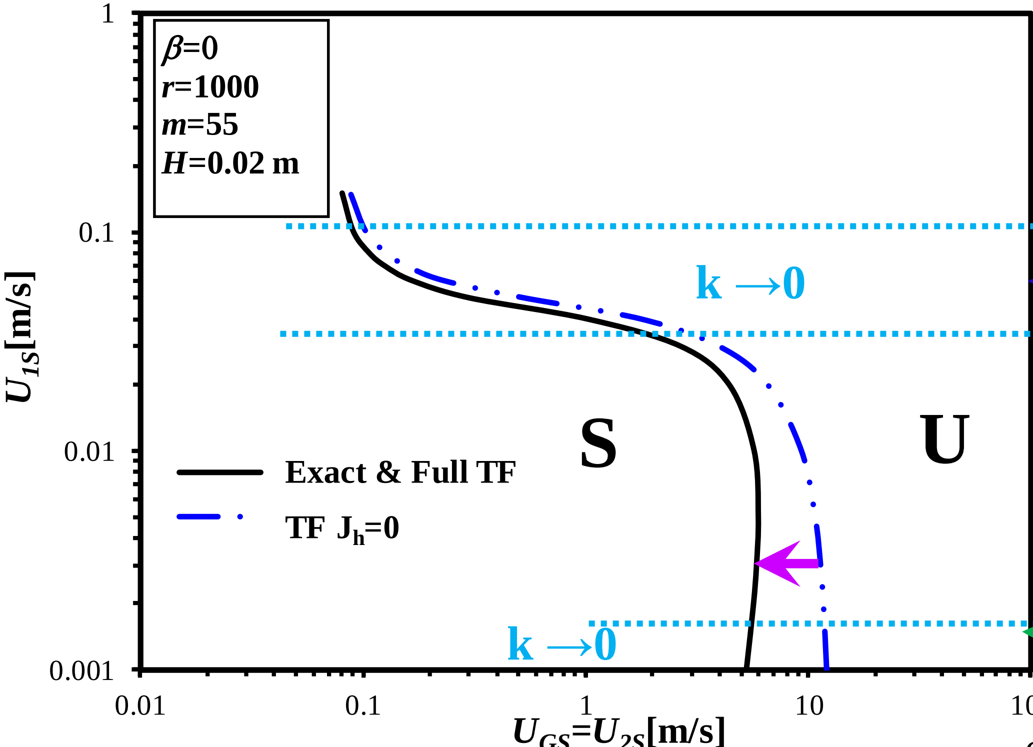
**$J_L$ dominant at low $U_{Gs}$ - Liquid-dominated KH mechanism determines the critical $U_{Ls}$**

**KH due to the Gas inertia is negligible!**

# Exact LW & TF model

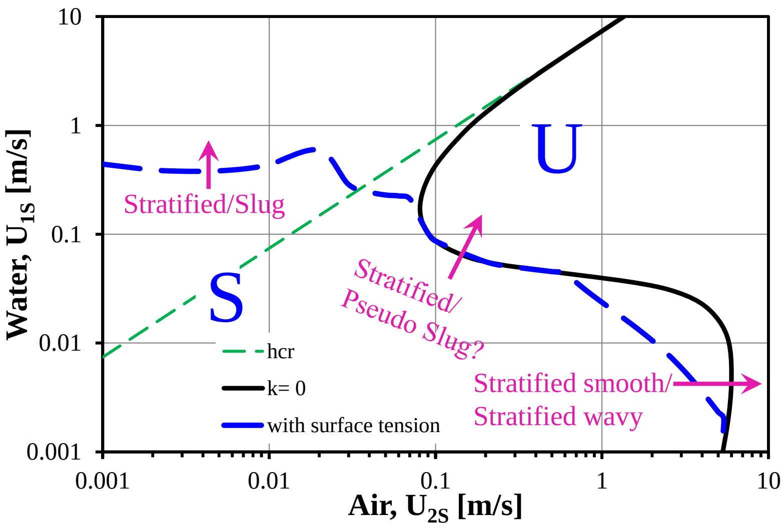
## Role of $J_1$, $J_2$, $J_h$

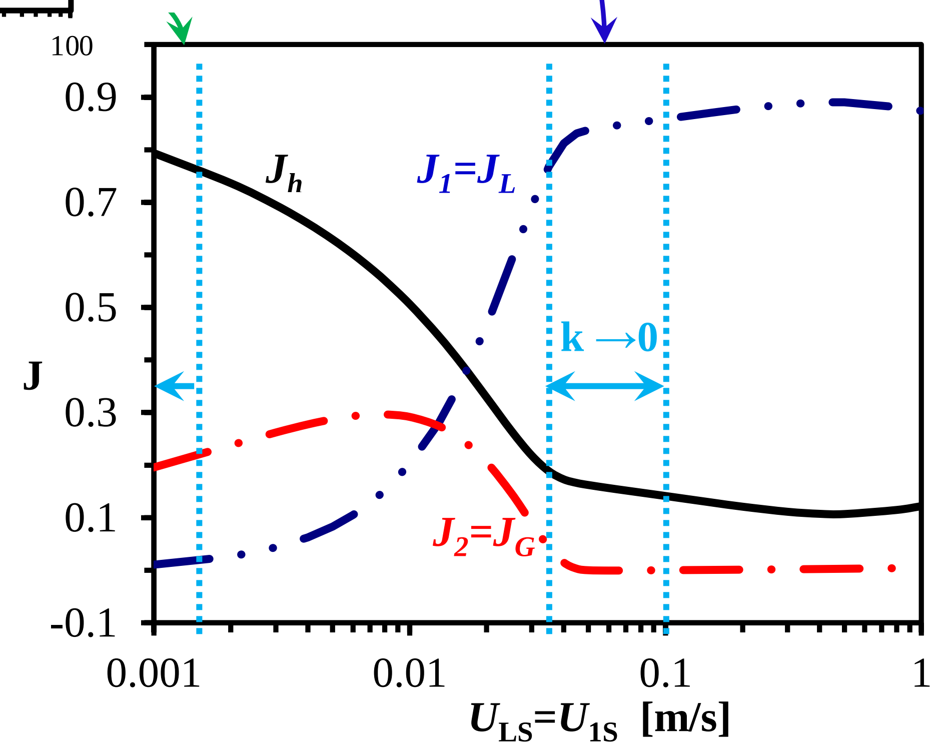
Horizontal air-water system, $H$=0.02m




KH mechanism
Liquid-dominated

'Sheltering' mechanism



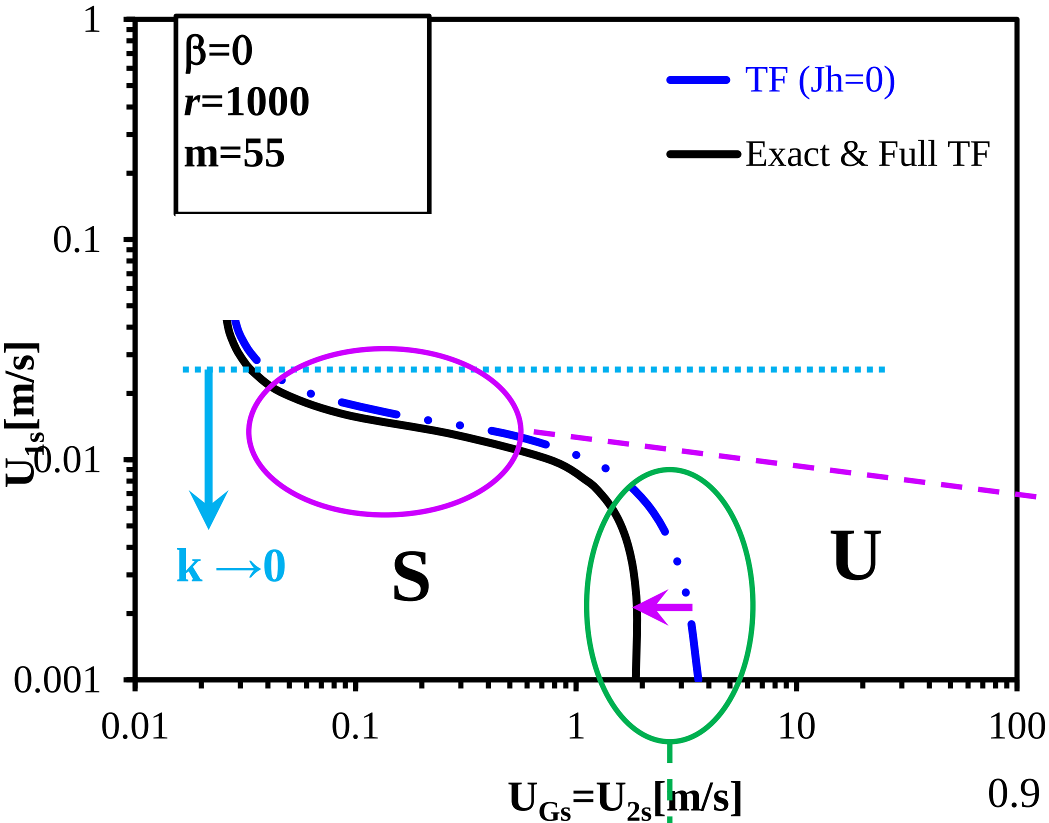



# Exact LW & TF model

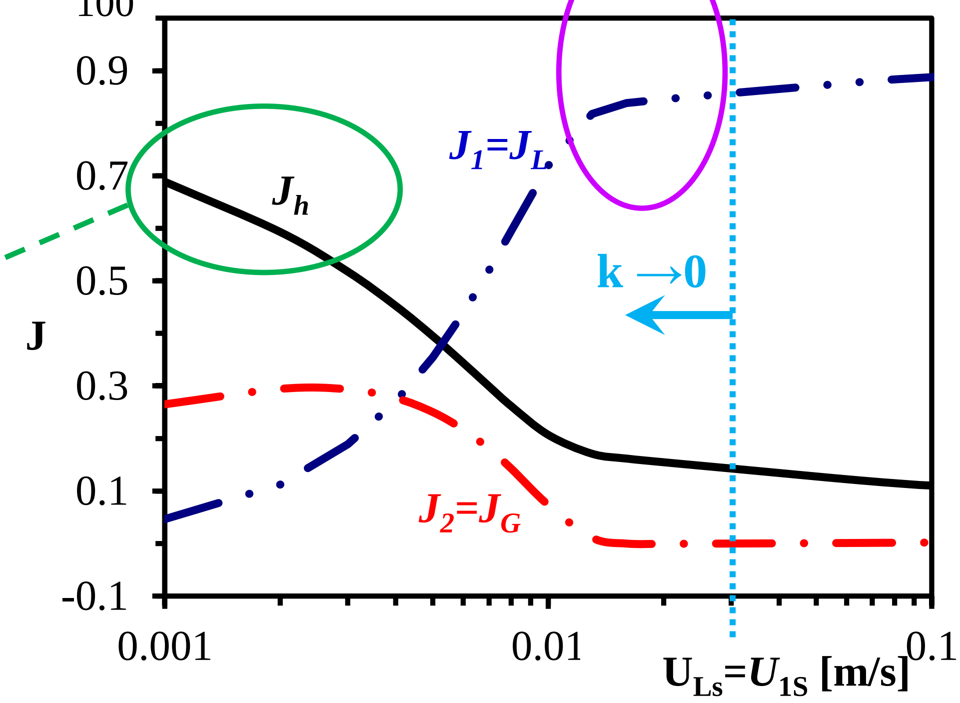
Horizontal air-water system

**Mini channel, $H$=0.002m**

## Role of $J_1$, $J_2$, $J_h$




**KH mechanism**
**Liquid-dominated**




**$J_h$ ("Sheltering" mechanism) is dominant, complimented by the gas inertia**

# Conclusions

- The exact solutions, although obtained for simple geometry, enable identifying the pertinent mechanisms, which are relevant also for the flow in realistic geometry (e.g., pipe flow), and are not represented in the commonly used two-fluid (TF) models.

- Closure relations that enable the TF model reproducing the exact steady state solution, as well as the exact Long Wave stability boundary for horizontal and inclined flows were identified.

- The “Sheltering” mechanism is important in triggering instability. Nevertheless, the contribution of the KH mechanism cannot be ignored, and definitely should not ruled out.

- Consideration of all wave number perturbations is essential for a correct prediction of the flow stability. Long wave stability analysis is generally insufficient- a severe limitation of TF models.

- Multiple holdups in inclined flows- the stability analysis indicates that more than one solution can be stable in the multiple solution regions of upward inclined and countercurrent flows.

# Thank you!

## Acknowledgment

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